Optimizing Charger Placement in E-Bike Sharing Systems with State-of-Charge Dynamics

Ghazaleh Mohseni Hosseinabadi*[1], Mehdi Nourinejad[1], and Peter Y. Park[1]

[1]Department of Civil Engineering, York University, Canada

Short summary

The growing adoption of electric bike-sharing systems introduces new challenges in managing bike availability due to state-of-charge (SOC) dynamics. Existing models typically treat bikes as homogeneous and do not capture the interaction between SOC evolution and system performance. This study addresses charger placement through a unified framework that integrates SOC dynamics, ridership estimation, and infrastructure deployment within a two-dimensional Markovian system. Steady-state probabilities are derived to estimate expected ridership, which serves as the objective in a station-level location optimization problem. A heuristic solution method combined with a single-pooling approximation ensures scalability for large networks. Results show that the proposed approach achieves near-optimal performance, within 0.5–1.5% of the global optimum. Increasing charger deployment improves availability and accessibility, although with diminishing returns. Case studies in Pittsburgh, Vancouver, and San Francisco demonstrate that SOC-aware charger placement significantly reduces low-SOC occurrences and adapts to different demand patterns.

Keywords: E-bike sharing systems, Charger placement, Ridership optimization, State-of-charge dynamics

1 Introduction

Shared micromobility systems have become a key component of sustainable urban transportation. Electric bikes (e-bikes), in particular, enhance accessibility by extending travel range and reducing physical effort, leading to rapidly increasing adoption. For example, e-bikes accounted for 46% of station-based trips in the U.S. in 2023 despite representing only one-third of the fleet (NACTO, 2024).

Unlike traditional bikes, e-bikes require frequent charging, introducing new operational and planning challenges. In station-based systems, managing state-of-charge (SOC) is critical for maintaining availability and service reliability. Low-SOC bikes are less attractive to users, while charging processes temporarily reduce supply. Since deploying chargers at all stations is often infeasible, determining their optimal placement becomes a central planning problem.

Existing bike-sharing models, including Markovian and queuing approaches, typically assume homogeneous bikes and do not capture SOC dynamics (Raviv & Kolka, 2013; Li et al., 2016; Kabra et al., 2020). Recent studies on charging infrastructure focus on system-level objectives but often rely on simplified or aspatial formulations (Song et al., 2024; Liu & Ouyang, 2025). As a result, the interaction between SOC dynamics, charging infrastructure, and network-level ridership remains insufficiently understood.

This study proposes a unified Markovian framework for SOC-aware charger placement in station-based e-bike systems. Each station is modeled as a two-dimensional Markov process capturing low- and high-SOC dynamics, while charger deployment is formulated as a location optimization problem that maximizes expected ridership.

To ensure scalability, we develop efficient solution methods, including approximation techniques that reduce the state space and a heuristic search for charger placement. The framework is evaluated through computational experiments and real-world case studies, demonstrating its effectiveness in improving availability and ridership across different urban contexts.

2 Methodology

We consider an e-bike sharing network with stations $\mathcal{S} = \{1, \ldots, S\}$, where each station s has capacity C_s. A subset $\mathcal{S}_n \subseteq \mathcal{S}$ of size N is equipped with chargers. We assume that all docks at charging stations support charging. The objective is to determine the optimal charger locations to maximize overall ridership, accounting for network flows and charging dynamics.

To capture battery dynamics while maintaining tractability, we model SOC using two states: low and high. Low-SOC bikes have limited battery, are less preferred for rental, and can be recharged when docked at charging stations. High-SOC bikes are fully or sufficiently charged, more attractive to users, and do not require charging. Upon return, a bike may remain in high-SOC or transition to low-SOC depending on trip conditions.

Markov Chain Model

We model each station $s \in \mathcal{S}$ as a Markov chain, with its state defined by the tuple (i_s, j_s), where i_s and j_s are the number of low- and high-SOC bikes at station s, respectively. The total number of bikes at a station must satisfy:

$$i_s + j_s \leq C_s, \quad 0 \leq i_s, j_s \leq C_s. \tag{1}$$

State transitions arise from arrivals, departures, and charging, each following Poisson processes.

Arrivals:

$$(i,j) \to (i+1,j) : \lambda_s^L, \quad (i,j) \to (i,j+1) : \lambda_s^H \tag{2}$$

Departures:

$$(i,j) \to (i-1,j) : \mu_s^L i, \quad (i,j) \to (i,j-1) : \mu_s^H j \tag{3}$$

Charging (if $s \in \mathcal{S}_n$):

$$(i,j) \to (i-1,j+1) : \gamma_s i \tag{4}$$

Each of the events results in a transition from one state to another. The probability of transitioning from state (i_s, j_s) to a neighboring state is proportional to the rate at which the corresponding event occurs. These transitions form a structured two-dimensional Markov process. Figure 1 illustrates the transition diagram for a station with a capacity of 3, providing a visual representation of how the system evolves.

The state space is defined as $\mathcal{K}_s = \{(i_s, j_s) \mid i_s + j_s \leq C_s\}$, $|\mathcal{K}_s| = \frac{(C_s+1)(C_s+2)}{2}$, which grows quadratically with capacity and exponentially increasing the complexity of the system as the state space expands with larger capacities.

Steady-state: Let $\pi_s(i,j)$ denote the stationary distribution. The system satisfies global balance and normalization as inflow into (i,j) = outflow from (i,j), $\sum_{(i,j)\in\mathcal{K}_s} \pi_s(i,j) = 1$. The steady-state distribution cannot be obtained in closed form except for very small capacities, because the two-dimensional birth–death structure induced by charging breaks the product form separability that underlies classical bike-sharing models. Therfore, $\pi_s(i,j)$ is obtained numerically by solving the global balance equations, implemented here using power iteration or Gaussian elimination.

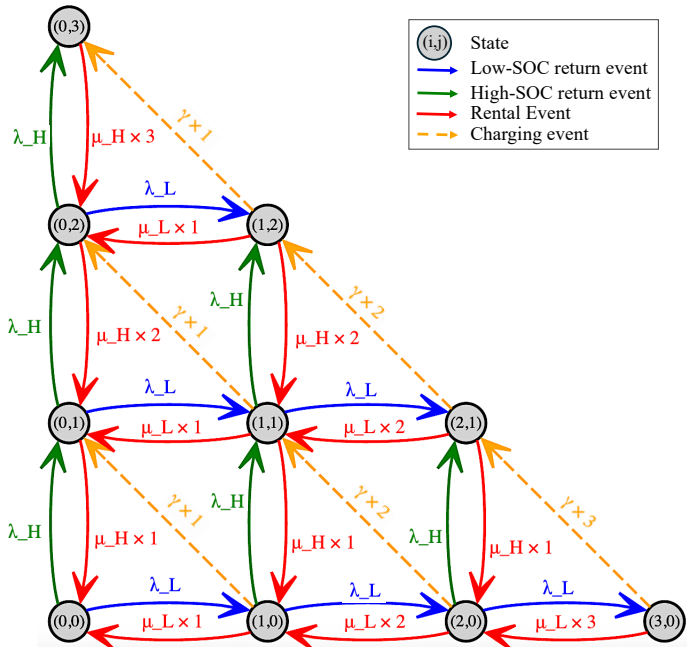


Figure 1: State transition diagram for a station: $C_s = 3$.

Optimization Model for Charger Placement

The objective is to maximize the total ridership across all stations, formulated in the mathematical optimization problem:

$$\text{Maximize} \sum_{s\in S}\Big(R_s^L + R_s^H\Big) \;=\; \sum_{s\in S}\sum_{(i,j)\in\mathcal{K}_s} \pi_s(i,j)\,\big(\mu_s^L\, i_s + \mu_s^H\, j_s\big) \tag{5}$$

$$\text{s.t} \quad \sum_{(i,j)\in\mathcal{K}_s} \pi_s(i,j) = 1, \quad \forall s \in S \tag{6}$$

$$\gamma_s \le x_s \gamma, \quad \forall s \in S \tag{7}$$

$$\sum_{s\in S} x_s = N \tag{8}$$

$$R_s^L = \sum_{(i,j)\in\mathcal{K}_s} \pi_s(i,j)\,\big(\mu_s^L\, i_s\big), \quad \forall s \in S \tag{9}$$

$$R_s^H = \sum_{(i,j)\in\mathcal{K}_s} \pi_s(i,j)\,\big(\mu_s^H\, j_s\big), \quad \forall s \in S \tag{10}$$

$$\lambda_t^L = \sum_{s\in S}\Big(R_s^L\,\theta_{s\to t}^{LL} + R_s^H\,\theta_{s\to t}^{HL}\Big), \quad \forall t \in S \tag{11}$$

$$\lambda_t^H = \sum_{s\in S}\Big(R_s^L\,\theta_{s\to t}^{LH} + R_s^H\,\theta_{s\to t}^{HH}\Big), \quad \forall t \in S \tag{12}$$

$$\sum_{(k,\ell)\in\mathcal{K}_s} \pi_s(k,\ell)\,T_s\big((k,\ell)\to(i,j)\big) = \pi_s(i,j) \sum_{(r,u)\in\mathcal{K}_s} T_s\big((i,j)\to(r,u)\big), \quad \forall (i,j)\in\mathcal{K}_s,\ \forall s\in S \tag{13}$$

$$T_s\big((i,j)\to(r,u)\big)=\begin{cases}\lambda_s^L, & \text{if } (r,u)=(i+1,j) \text{ and } i+j<C_s\\ \lambda_s^H, & \text{if } (r,u)=(i,j+1) \text{ and } i+j<C_s\\ \mu_s^L\, i, & \text{if } (r,u)=(i-1,j) \text{ and } i\geq 1\\ \mu_s^H\, j, & \text{if } (r,u)=(i,j-1) \text{ and } j\geq 1\\ \gamma_s\, i, & \text{if } (r,u)=(i-1,j+1) \text{ and } i\geq 1, (i-1)+(j+1)\leq C_s\\ 0, & \text{otherwise}\end{cases} \qquad \forall s\in S \tag{14}$$

$$\pi_s(i,j)\geq 0, \quad \lambda_s^L,\lambda_s^H\geq 0, \quad x_s\in\{0,1\}, \quad \forall (i,j)\in\mathcal{K}_s,\ \forall s\in S \tag{15}$$

The optimization objective aims to maximize the total expected ridership across all stations. The ridership is captured by the expected outflows of low- and high-SOC bikes. To ensure that the Markov model maintains valid probability distributions, Constraint (6) enforces that the sum of steady-state probabilities equals one at each station. We introduce a binary decision variable $x_s \in \{0,1\}$ to indicate whether station s is equipped with a charger. If $x_s = 1$, low-SOC bikes recharge to high-SOC at a rate γ (Constraint (7)). Constraint (8) restricts the number of stations that can be equipped with chargers to exactly N.

To accurately model the movement of bikes between stations while considering variations in their SOC, we assume that the expected outflows at each station are determined by the SOC transition matrix and the origin-destination (O-D) flow distribution. We define:

$$\theta_{s\to t}^{XY} = Q_{s\to t}\, P_s^{XY}, \quad \forall s,t\in S,\ \forall XY\in\{LL,LH,HL,HH\}, \tag{16}$$

where $Q_{s\to t}$ represents the O-D fraction, indicating the proportion of bikes rented from station s that are ultimately returned to station t. The term P_s^{XY} is drived from the SOC transition matrix as below:

$$P_s^{XY} = \begin{pmatrix} p_s^{LL} & p_s^{LH} \\ p_s^{HL} & p_s^{HH} \end{pmatrix}, \tag{17}$$

where each entry of P_s^{XY} represent the probability a bike transitioning from SOC state X to state Y while traveling. The rental dynamics of bikes at each station are governed by Constraint (9) and (10), which quantify the expected outflow of low-SOC and high-SOC bikes, respectively. Constraint (11) and (12) ensure that each station receives an appropriate number of incoming bikes, calculated based on outgoing flows from other stations and transition probabilities of each pair of stations. The fundamental principle and global balance equations of steady-state equilibrium is captured by Constraint (13), which ensures that the probability distribution at each station remains consistent. Constraint (14), defines the transition rate function and specifies how bikes move between low- and high-SOC states due to events.

This optimization problem poses significant computational challenges due to several key factors. First, the Markovian state representation at each station introduces a large state space, as the number of possible system states grows exponentially with capacity. Second, the model incorporates nonlinear interdependencies between stations, particularly through the SOC transition dynamics and the coupling of inflows and outflows across the network. Additionally, network-wide coupling in bike availability means that decisions regarding charger placements and station capacities impact the entire system, further increasing the problem's dimensionality and computational burden. Due to these challenges, solving the problem exactly using traditional optimization solvers may be computationally infeasible for large-scale systems, motivating the use of heuristic approaches.

3 Solution Methodology

Ridership Equilibrium for a Fixed Charger Configuration

For a fixed set of charging stations $S_n \subseteq S$, we compute the equilibrium inflows $(\lambda_s^L, \lambda_s^H)$ and steady-state probabilities $\pi_s(i,j)$. The solution is obtained through an iterative fixed-point procedure. Starting from an initial guess of inflows, at each iteration we:

- compute station-level steady-state probabilities $\pi_s(i,j)$ given current inflows,
- evaluate expected outflows R_s^L and R_s^H,
- update inflows using OD-SOC redistribution across the network.

To ensure convergence, we apply the Method of Successive Averages (MSA):

$$\begin{cases} \lambda_t^{L,m} = \beta_m \widehat{\lambda}_t^{L,m} + (1-\beta_m)\lambda_t^{L,(m-1)}, \\ \lambda_t^{H,m} = \beta_m \widehat{\lambda}_t^{H,m} + (1-\beta_m)\lambda_t^{H,(m-1)}, \end{cases} \qquad \beta_m = \frac{1}{m}. \tag{18}$$

The process continues until convergence of inflows. The resulting steady-state probabilities and flows define the ridership equilibrium for the given charger configuration, with total ridership:

$$\mathcal{R} = \sum_{s \in S} \left(R_s^L + R_s^H\right). \tag{19}$$

This procedure captures the network-wide coupling between station dynamics and bike flows, and yields a consistent equilibrium solution for any fixed charger layout.

Simplification Method for Large-Scale Systems-Single-Pooling State Approximation

For large station capacities, the two-dimensional state space becomes computationally intractable ($O(C_s^2)$). To address this, we approximate the system by tracking only the total number of bikes $k = i + j$, reducing the model to a one-dimensional $M/M/1/C$ process. Let α denote the fraction of low-SOC bikes. The effective service rate is:

$$\mu(k) = k\left[\alpha\mu_s^L + (1-\alpha)\mu_s^H\right] = k\nu, \tag{20}$$

where $\nu = \alpha\mu_s^L + (1-\alpha)\mu_s^H$, and total arrival rate $\lambda = \lambda_s^L + \lambda_s^H$. The utilization factor is $\rho = \frac{\lambda}{\nu}$ and the steady-state distribution follows $\pi_s(k) = \begin{cases} \frac{\rho^k}{\sum_{m=0}^{C_s}\rho^m}, & \rho \neq 1, \\ \frac{1}{C_s+1}, & \rho = 1. \end{cases}$ The fraction α is obtained from flow balance as $\alpha = \frac{\lambda_s^L}{(\mu_s^L+\gamma)E[k]}$, where $E[k] = \sum_{k=0}^{C_s} k\,\pi_s(k)$. An iterative procedure updates α until convergence. The joint distribution is then approximated using $\pi_s(i,j) \approx \pi_s(k)\binom{k}{i}\alpha^i(1-\alpha)^j$, where $i + j = k$. This approximation reduces complexity from $O(C_s^2)$ to $O(C_s)$ while preserving key system dynamics.

Ridership Optimization: Heuristic Charger Placement

With the equilibrium solver established, we now seek an effective way to select which N stations should host chargers in order to maximize overall ridership. We implement a local-improvement or "swap" heuristic. Starting from an initial solution, charger-equipped and non-equipped stations are iteratively exchanged to improve total ridership. Stations are evaluated using three scoring metrics:

Direct Ridership Improvement (DRI):

$$\Delta\mathcal{R}_s = \frac{\partial}{\partial\gamma_s}\sum_{s\in S}(R_s^L + R_s^H) \approx \mathcal{R}(\gamma_s = \gamma_{\text{charging}}) - \mathcal{R}(\gamma_s = 0), \tag{21}$$

Low-SOC Prevalence:

$$\text{Score}_s^{(A)} = \sum_{\substack{(i,j)\in\mathcal{K}_s \\ i\geq 1}} \pi_s(i,j) \cdot \frac{R_s^L}{R_s^L + R_s^H}, \tag{22}$$

Charging Potential:

$$\text{Score}_s^{(B)} = \frac{\gamma \sum_{(i,j)\in\mathcal{K}_s,\, i\geq 1} i\, \pi_s^{(\text{charged})}(i,j)}{R_s^L + R_s^H}. \tag{23}$$

At each iteration, chargers are reallocated by swapping low-performing charging stations with high-scoring candidates. The equilibrium is recomputed after each swap, and updates are accepted only if total ridership increases. The procedure terminates when no improving swap exists, yielding a locally optimal charger configuration.

4 Results

Computation Example with Exact Probability Model

To evaluate the proposed framework, we consider a computational example on a test network. Table 1 shows that all heuristic methods achieve near-optimal solutions with significantly lower computation times. The DRI approach performs best, with a negligible ridership gap (0.04) while reducing runtime from over 1000s to 11s. The other methods exhibit slightly larger gaps (0.13) but offer further computational savings. Figure 2 illustrates ridership improvement across iterations. DRI (blue) converges fastest and reaches the highest ridership, closely matching the brute-force optimum. Charging Potential (green) performs moderately, while Low-SOC Prevalence (orange) converges more slowly. All heuristics improve ridership over iterations, with differences in convergence speed and final performance. These results demonstrate that the proposed methods provide an efficient and scalable alternative to brute-force optimization with minimal loss in solution quality.

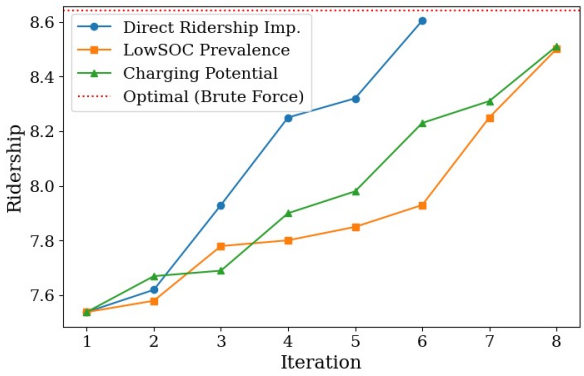


Figure 2: Ridership trend for different approaches.

Table 1: Comparison of Optimal Configurations, Ridership, and Computation Time.

Method	Optimal Config.	Ridership	Computation Time (s)
Direct Ridership Improvement	(7, 10, 13, 15, 18)	8.60	11.24
Low-SOC Prevalence	(5, 6, 10, 13, 15)	8.51	6.72
Charging Potential	(5, 6, 10, 13, 15)	8.51	2.25
Brute-Force	(4, 5, 7, 10, 15)	8.64	1032.11

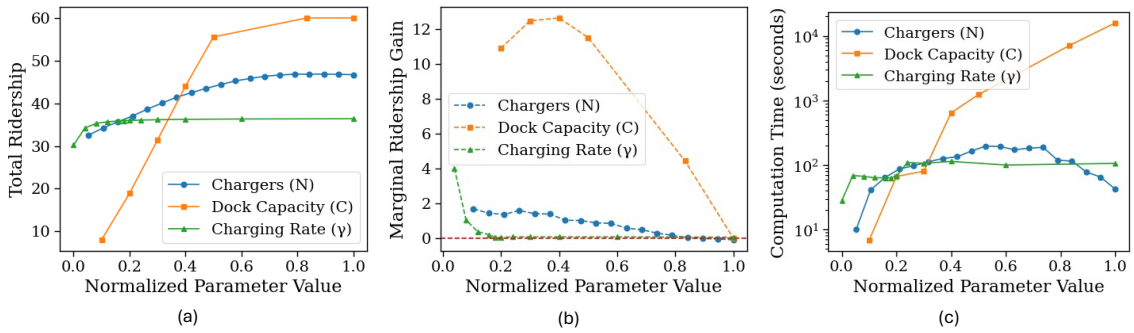


Figure 3: Comparative impact of key system parameters on ridership, marginal gains, and computational cost.

To assess robustness, we conduct a sensitivity analysis by varying three parameters: number of charging stations (N), station capacity (C_s), and charging rate (γ). Figure 3 summarizes the results. Increasing C_s yields the largest improvements in ridership, followed by N, while γ exhibits rapid saturation. All parameters show diminishing marginal returns, with the sharpest decline observed for γ. In terms of computational cost, C_s has the most significant impact due to state space growth, followed by N, while γ has minimal effect. These results highlight the importance of balanced infrastructure design, where capacity and charger placement play a more critical role than charging speed in improving long-run system performance.

Comparison of Exact Probability vs. Single-Pooling State Approximation Model

We compare the exact probability model with the single-pooling approximation to evaluate scalability and accuracy. Table 2 shows that both models produce nearly identical charger configurations and ridership values across all capacities. However, computational differences are substantial. As capacity increases, the exact model becomes computationally expensive (e.g., over 4000s at $C = 30$), while the approximation remains efficient (77s), achieving more than 50× speedup.

Table 2: Comparison of exact probability vs. single-pooling state approximation model

Model	Metric	$C = 3$	$C = 4$	$C = 10$	$C = 15$	$C = 30$
Exact Prob.	Optimal Config.	(0, 2, 5)	(0, 5, 6)	(0, 2, 5)	(0, 2, 9)	(0, 2, 9)
	Ridership	4.28	6.22	19.67	29.96	30.00
	Computation Time (s)	1.21	2.62	17.43	673.10	4086.10
Single-Pool Approx.	Optimal Config.	(0, 2, 5)	(0, 5, 6)	(0, 2, 5)	(0, 2, 9)	(0, 2, 9)
	Ridership	4.06	6.18	19.29	29.55	30.00
	Computation Time (s)	4.10	3.47	27.57	38.03	77.24
Brute-Force	Optimal Config.	(0, 5, 6)	(0, 5, 6)	(0, 2, 5)	(0, 6, 9)	(0, 6, 9)
	Ridership	4.28	6.22	19.67	29.97	30.01
	Computation Time (s)	3.20	5.23	52.42	1830.02	16271.57

Figure 4 further illustrates that the approximation closely tracks the exact model in terms of ridership while significantly reducing runtime. The single-pooling approximation provides an effective trade-off between accuracy and computational efficiency, making it suitable for large-scale applications.

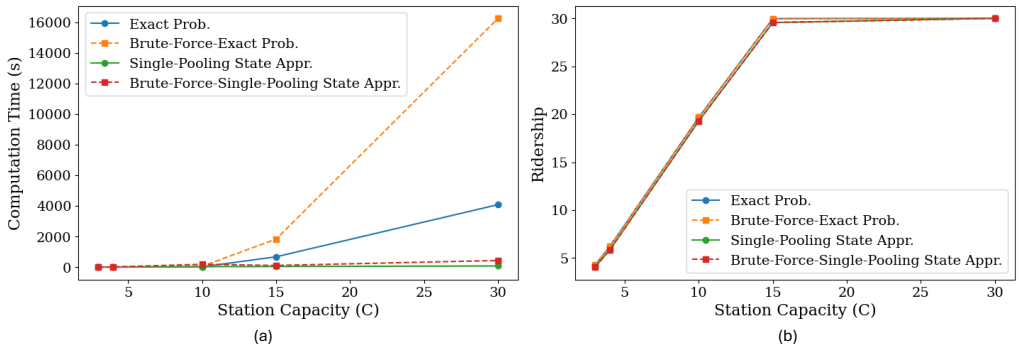


Figure 4: Exact and approximation models comparison in terms of station capacity and a)Computation time, b) Ridership

5 Case Study

We evaluate the proposed framework on real-world bike-sharing systems of different scales. Table 3 summarizes the results under varying levels of charger deployment.

Table 3: Steady-state performance across cities and electrification levels.

Scenario	**High-SOC Departures (%)**	**SOC Shortage Probability**	**Chronic Low-SOC Stations**
Pittsburgh (58 Stations)			
10% chargers	54	0.12	16
30% chargers	68	0.08	10
50% chargers	78	0.04	4
Vancouver (257 Stations)			
10% chargers	47	0.19	40
50% chargers	63	0.07	18
San Francisco (331 Stations)			
10% chargers	35	0.28	78
50% chargers	57	0.15	42

Across all cities, increasing charger coverage significantly improves system performance, as reflected by higher shares of high-SOC departures and reduced shortage probabilities. The most pronounced improvements occur in larger and more constrained systems, where electrification plays a critical role in maintaining availability. Figure 5 illustrates the resulting charger placement patterns, highlighting the prioritization of high-demand and central stations. Overall, the results demonstrate that the proposed approach effectively improves SOC availability and system reliability across diverse urban settings.

6 Conclusions

This study presents a unified optimization framework for charger placement in station-based e-bike sharing systems that explicitly incorporates state-of-charge (SOC) dynamics within a Markovian structure. By capturing the interaction between charging infrastructure, bike availability, and ridership, the framework enables system-level evaluation of charging strategies.

Results show that the proposed approach achieves near-optimal performance, within 0.5–1.5% of the global optimum, while significantly reducing computation time. The Direct Ridership Improvement heuristic consistently delivers the fastest convergence and best per-

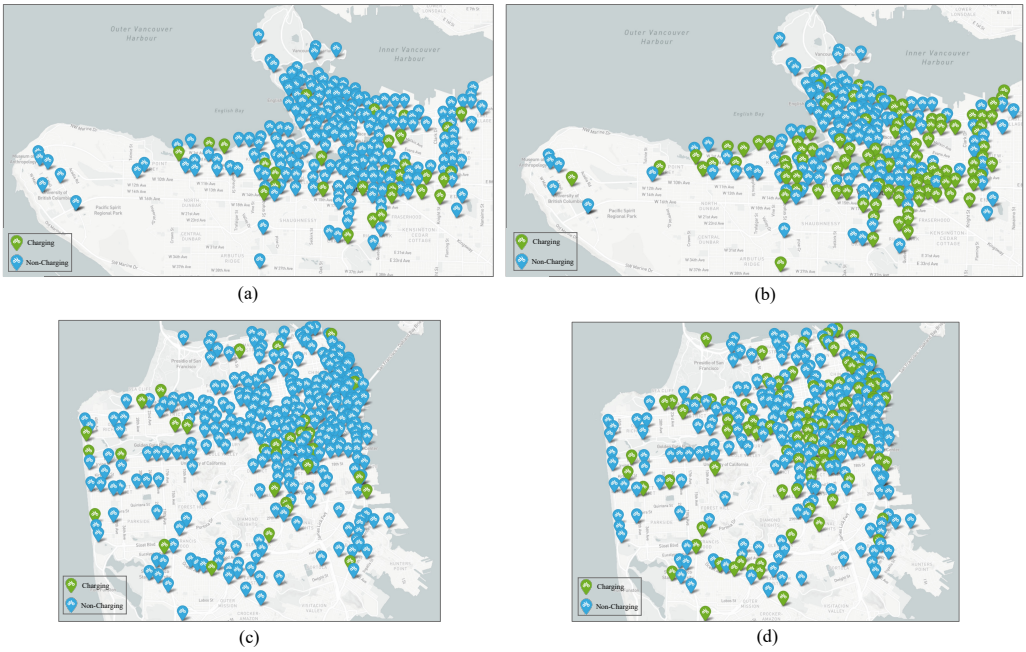


Figure 5: Optimal charging-station configurations for: (a) Vancouver at 10% electrification, (b) Vancouver at 50%, (c) San Francisco at 10%, and (d) San Francisco at 50%.

formance. Sensitivity analyses reveal diminishing returns in ridership gains as charging infrastructure expands, emphasizing the importance of strategic deployment.
The framework scales effectively to large networks and performs consistently across case studies in Pittsburgh, Vancouver, and San Francisco, providing practical guidance for SOC-aware infrastructure planning in shared mobility systems.

References

Kabra, A., Belavina, E., & Girotra, K. (2020). Bike-share systems: Accessibility and availability. *Management Science*, *66*(9), 3803–3824.

Li, Q.-L., Chen, C., Fan, R.-N., Xu, L., & Ma, J.-Y. (2016). Queueing analysis of a large-scale bike sharing system through mean-field theory. *arXiv preprint arXiv:1603.09560*.

Liu, Y., & Ouyang, Y. (2025). Planning charging stations for mixed docked and dockless operations of shared electric micromobility systems. *Transportation Research Part C: Emerging Technologies*, *171*, 104989.

NACTO. (2024). *Shared micromobility in 2023.* Retrieved from `https://nacto.org/publication/shared-micromobility-in-2023/`

Raviv, T., & Kolka, O. (2013). Optimal inventory management of a bike-sharing station. *Iie Transactions*, *45*(10), 1077–1093.

Song, J., Li, B., Szeto, W., & Zhan, X. (2024). A station location design problem in a bike-sharing system with both conventional and electric shared bikes considering bike users' roaming delay costs. *Transportation research part E: logistics and transportation review*, *181*, 103350.