User equilibrium calculation in Multimodal static assignment using MFG

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Short summary

The object of the paper is to analyze the calculation of user equilibria in static multimodal networks. The multimodal model is a static version of the GSOM (generic second order model) which is a macroscopic model in which passenger and vehicular flows are distinct but the passenger flow is subordinate to the vehicular flow. Nodes in the network represent intersections, stations, interchanges and hubs. The paper will first address the conception of the static multimodal GSOM model, in particular the issue of travel costs. Then the paper will consider progressive travel choice of travelers, which is described as a probability of route choice. Travelers improve progressively their choice in order to achieve optimal route choice while minimizing their daily rate of choice change. This process is described as a MFG (mean field game) problem, a solution of which is proposed.
Keywords: GSOM multimodal model; User equilibrium; Static networks; Mean field games; Stochastic choice model

1 Introduction

Static assignment models remain very useful for transportation planning authorities, and thus require new progress. In particular multimodal models need to be improved Jiang & Nielsen (2022); Alessandretti et al. (2023); Fuchs & Wong (2026); Montella et al. (2026). The aim of this paper is to address the classical traffic assignment model under a new angle based on the MFG (mean field games) technique. MFG are based on the classical concept of mean fields, developped in statistical mechanics. This classical concept for many particle systems has been combined in recent work Lasry & Lions (2007); Achdou et al. (2012) with the idea of optimal control applied on the dynamics of the individual particles. Thus MFG theory is mainly concerned with the equilibrium of such systems.
This idea has been applied to static assignment in several works such as Achdou et al. (2012); Benamou et al. (2017); Salhab et al. (2018); Ong (2024); Achdou et al. (2024). The reference Xuan et al. (2025) provides an overview. Most approaches concentrate on the route choice, i.e. they aim to optimize the trajectory of travelers in the network, precisely their successive decisions at nodes. Interestingly the reference Benamou et al. (2017) proposes a 2D approach (a continuum of travelers) but the approach does not apply to multimodal transportation networks, as it leads to a unique optimum, which is not the case with multimodal networks Ameli, Lebacque, & Leclercq (2022).
The idea developped in this paper stems from Ameli, Faradonbeh, et al. (2022); Khoshyaran & Lebacque (2027). Rather than assuming a route choice based on utility functions, we consider that individual travelers adapt their route choice probability on a day to day basis in view of travel conditions resulting from the choices of other travelers. The main assumtion is that travelers aim at smooth changes in their choice probabilities and at minimizing their expected travel costs. The supporting traffic model is adapted from the multi-modal GSOM modelLebacque & Khoshyaran (2018a,b).

2 Methodology

Derivation of a GSOM static multimodal model.

General concepts

The model is static and multimodal, derived from the multimodal version of the GSOM Lebacque & Khoshyaran (2018a,b). The main features of the multimodal GSOM can be summarized as follows: the model is macroscopic, with two flows (vehicular and passenger), both of which are conserved, the passenger flow being advected by the vehicular flow. The vehicular flow is determined by a conservation equation combined with a fundamental diagram. Both vehicular and passenger flows admit attributes which are also advected. The main attributes considered in this paper are the routes, but many other attributes may be relevant. Nodes in the network represent, depending on position: intersections, station, interchanges, multimodal poles. Given that the assignment problem will be treated by the MFG technique we adopt a path flow representation, as well as some other representations which will be introduced progressively.

Flows

Let $\mathcal{W}$ be the set of OD (origin-destination) couples for passengers. The same set is used for vehicles. For each OD couple $w \in \mathcal{W}$, the notation $\mathcal{P}_w$ refers to the set of traveler paths joining w, and $\mathcal{Q}_w$ refers to the set of vehicular paths joining w. We denote $\mathcal{P} \stackrel{def}{=} \cup_{w\in\mathcal{W}}\mathcal{P}_w$ and $\mathcal{Q} \stackrel{def}{=} \cup_{w\in\mathcal{W}}\mathcal{Q}_w$ the sets of all traveler respectively vehicular flows. Vehicular paths are modal, the same rule applies to arcs. $\mathcal{A}$ denotes the set of arcs in the network. A traveler path $p \in \mathcal{P}$ may include several vehicular paths $q \in \mathcal{Q}$. Let us consider an arc $a \in \mathcal{A}$: this arc carries a passenger flow P_a and a vehicular flow Q_a, with the ratio $\varpi_a \stackrel{def}{=} P_a/Q_a$ denoting the passenger load of vehicles on arc a. The arc flows are constrained as follows:

$$\left| \begin{array}{ll} P_a \leq Q_a\, \varpi_{a,max} & (1.a) \\ Q_a \leq Q_{a,max} & (1.b) \end{array} \right. \tag{1}$$

with $\varpi_{a,max}$ the capacity of vehicles which bounds the passenger load, and $Q_{a,max}$ the capacity of the arc a. The arc and path flows are connected by the following:

$$\left| \begin{array}{l} P_a = \sum_{p\in\mathcal{P},p\ni a} f_p \\ Q_a = \sum_{q\in\mathcal{Q},q\ni a} g_q \end{array} \right. \tag{2}$$

where f_p denotes the traveler flow on $p \in \mathcal{P}$ and g_q denotes the vehicular flow on path $q \in \mathcal{Q}$. Typically most vehicular flows corresponding to public transportation (such as trains or trams, possibly buses) are given and therefore constrain passenger flows via (1.a). Conversely, passenger flows induce vehicular flows in the case of private cars or on-demand road modes (via occupancy), which impacts their perceived costs and also limits passenger flows by the conjunction of (1.a) and (1.b). In the paper we simply consider generic link flow constraints such as

$$P_a \leq P_{a,max} \quad \forall a \in \mathcal{A} \tag{3}$$

regardless of the actual formation of the capacity constraint. We will also consider mainly passenger flows. Let us notice that the conservation of flows results from their structure (2).

Costs

The details of the evaluation of costs are beyond the scope of this abstract. Let us state the main principles.

1. Costs are calculated for traveler paths $p \in \mathcal{P}$; the cost of such a path is denoted Γ_p. All path costs may include fares or tolls. The cost of a path is equal to the sum of costs of the arcs and nodes comprising the path;
2. For arcs $a \in \mathcal{A}$ belonging to private vehicle sub-paths of passenger, the cost is evaluated based on the total vehicle flow Q_a. The flow Q_a is given by (2.b). In the case of private vehicles or light on-demand solutions, the flow g_p results from the corresponding passenger flow f_p by application of a mean occupancy. Thus Q_a is expressed as a combination of flows f_p, with $p \in \mathcal{W}, p \ni a$. The cost of Q_a results from the fundamental diagram of the underlying GSOM model;

3. Let us consider an arc $a \in \mathcal{A}$ belonging to transyt sub-paths. The vehicular flow on such arcs is given, thus their cost is assumed constant, subject to possible congestion effects (vehicular congestion due to passenger crowding on platforms, or congestion in carriages Leurent (2011));
4. Nodes in the GSOM model are considered as pointwise or multi-buffers, depending on the infrastructure. Recapturing directly the time spent in buffers in a static model can be effectuated by using a modified fundamental diagram as developped for bounded acceleration models Lebacque (2003); Srivastava et al. (2015). In the case of multimodal poles with pedestrian components constant travel times suffice.
5. Capacity constraints such as (1) are very important and difficult to take into account. Thus they have given rise to a vast literature, for instance Smith et al. (2019); Bliemer & Raadsen (2020); Fu et al. (2025); Shi et al. (2025). Here these capacity constraints will be taken into account by penalty functions applying to costs. The modified fundamental diagram mentionned in the previous item 4 might also be used.

Equilibrium

The equilibrium is considered for path flows $f \overset{def}{=} (f_p)_{p\in\mathcal{P}}$ only. The previous subsubsection shows that the path costs are expressed in terms of the path flows, i.e. $\Gamma_p(f)$. The equilibrium conditions for the traveler path flows is expressed by Wardrop's first principle:

$$(UE^w) \quad \left| \begin{array}{ll} \sum_{p\in\mathcal{P}_w} (\Gamma_p(f) - \Gamma^{w,*}) \cdot f_p = 0 & (4.a) \\ \min_{r\in\mathcal{P}_w} \Gamma_p(f) = \Gamma^{w,*}(f) & (4.b) \end{array} \right. \tag{4}$$

for all $w \in \mathcal{W}$. The flow f satisfies

$$(\mathcal{K}) \quad \left| \begin{array}{ll} D_w = \sum_{p\in\mathcal{P}_w} f_p \quad \forall w \in \mathcal{W} & (5.a) \\ f_p \geq 0 \quad \forall p \in \mathcal{P} & (5.b) \end{array} \right. \tag{5}$$

Let us denote by $\mathbf{P}_\mathcal{K}$ the projector on the closed bounded convex set $(\mathcal{K})$ defined by (5), and let us define $\Gamma(f) \overset{def}{=} (\Gamma_p(f))_{p\in\mathcal{P}}$. Then the equilibrium conditions (4) can be expressed as a fixed point problem:

$$f = \mathbf{P}_\mathcal{K}\left[f - \Gamma(f)\right] \quad \forall \varsigma > 0 \tag{6}$$

which, given mild regularity conditions on the application $f \to \Gamma(f)$ (continuity), yields the existence of an equilibrium by a standard Brouwer type fixed point argument. In the multimodal case the equilibrium needs not be unique.

Mean-field game approach to static assignment.

Definitions

Let us denote x a generic traveler, $\mathcal{X}_w$ the set of travelers of OD $w \in \mathcal{W}$ traveling during a period of time of reference. Thus the number of travelers $|\mathcal{X}_w|$ is equal to the demand D_w.
The idea of the method developped in this subsection is that each traveler adjusts his travel choices progressively, from day to day, in order to minimize his expected travel cost. Let us define:

- π_q^x the probability for traveler $x \in \mathcal{X}_w$ to chose route $q \in \mathcal{P}_w$ and $\pi^x \overset{def}{=} \left(\pi_q^x\right)_{q\in\mathcal{P}}$ the vector of route choice probabilities;
- $\mathbf{I}^x$ denotes a random variable which can take the values $J_p, p \in \mathcal{P}_w$, with the probability π_p^x, where the vector J_p admits the components

$$J_{p,q} = \left[\begin{array}{lll} 1 & \text{if} & p = q \\ 0 & \text{if} & p \neq q \end{array} \right.$$

The mmean of $\mathbf{I}^x$ is given by $\mathbb{E}(\mathbf{I}^x) = \pi^x$ and its variance $\mathbf{V}(\pi^x)$ is given by

$$\mathbf{V}(\pi^x) = \mathbb{E}\left(\mathbf{I}^x \otimes \mathbf{I}^x\right) - \mathbb{E}\left(\mathbf{I}^x\right) \otimes \mathbb{E}\left(\mathbf{I}^x\right)$$

Thus for all $x \in \mathcal{X}_w$ and all $p, q \in \mathcal{P}_w$,

$$\mathbf{V}_{pq}(\pi^x) = \left[\begin{array}{lll} \pi_p^x - {\pi_p^x}^2 & \text{if} & p = q \\ -\pi_p^x\, \pi_q^x & \text{if} & p \neq q \end{array} \right.$$

The random variables $\mathbf{I}^x$ are assumed to be independent. Finally let us note that the flow on route $p \in \mathcal{P}_w$ is given by $f_p = \sum\limits_{x\in\mathcal{X}_w} \mathbf{I}_q^x$. By construction: $\sum\limits_{x\in\mathcal{X}_w} \mathbf{I}^x = D_w$.

Optimal traveler trajectories

Consider a traveler $x \in \mathcal{X}_w$ adjusting progressively his route choice π^x over a period $\tau \in [t_0, t_0+T]$. τ might be considered as time in a day-to-day choice process. We assume that the traveler will aim to minimize his expected travel cost $\sum_{q\in\mathcal{P}_w} \left[\pi_q^x(\tau) G_q\left(\pi(\tau)\right)\right]$ while minimizing his rate of change of his choice probabilites. Thus we formulate the optimal trajectory problem for a traveler $x \in \mathcal{X}_w$ as:

$$U^w\left(t_0, \pi^x_{init}\right) \stackrel{def}{=} \min_{v^x(.)} \int_{t_0}^{t_0+T} \left[|v^x(\tau)|^2/2A + \exp(\vartheta\tau) \sum_{q\in\mathcal{P}_w} \left(\pi_q^x(\tau) G_q\left(\pi^x(\tau)\right)\right) \right] d\tau$$
$$\left| \begin{array}{ll} \frac{d}{d\tau}\pi_q^x(\tau) = \pi_q^x(\tau)\, v_q^x(\tau) \quad \forall q \in \mathcal{P}_w & (7.a) \\ \sum_{q\in\mathcal{P}_w} \pi_q^x(\tau)\, v_q^x(\tau) = 0 & (7.b) \\ \pi^x(\tau)_{|\tau=t_0} = \pi^x_{init} & (7.c) \end{array} \right. \tag{7}$$

The relative importance given to the expected travel cost with respect to the rate of change increases exponentially with day τ, hence the factor $\exp(\vartheta\tau)$ weighing the travel cost. Thus over a long enough optimization period the traveler minimizes mainly his travel cost. U^w denotes the value function of the control problem (7). The formulation of (7.a) guaranties that if π^x_{init} is positive it stays positive at all times τ, and (7.b) implies that $\sum_{q\in\mathcal{P}_w} \pi_q^x(\tau) = 1$ stays equal to 1 at all times τ. The formulation (7) ensures that if the initial condition π^x_{init} is a probability distribution, $\pi^x(\tau)$ will be a probability distribution at all times τ. A denotes a normalization coefficient, and $\sum_{q\in\mathcal{P}_w} \left(\pi_q^x(\tau) G_q\left(\pi^x(\tau)\right)\right)_{|\tau=t_0+T}$ the terminal criterion which is the expected travel cost of traveler $x \in \mathcal{X}_w$.

We introduce $R = (R_q)_{q\in\mathcal{P}_w}$ the dual variable of the dynamic system constraint (7.a), and for convenience, we introduce a diagonal matrix $\overline{R}$ whose diagonal elements are the elements of R, i.e. $\overline{R}_{qq} = R_q \; \forall q \in \mathcal{P}_w$. The hamiltonian of (7) results:

$$H(\tau, \pi, v, R) \stackrel{def}{=} \frac{|v|^2}{2A} + \pi.\overline{R}.v + \exp(\vartheta\tau) \sum_{q\in\mathcal{P}_w} \left(\pi_q G_q\left(\pi\right)\right) \tag{8}$$

where v satisfies (7.b) i.e. $v.\pi = 0$. The reduced hamiltonian $H_0(\pi, R)$ results by minimizing H with respect to v satisfying $v.\pi = 0$. Hence:

$$H_0(\tau, \pi, R) = \min_{v/v.\pi=0} H(\tau, \pi, v, R) = \frac{A}{2}\left(-|\overline{R}.\pi|^2 + \frac{(\pi.\overline{R}.\pi)^2}{|\pi|^2} \right) + \exp(\vartheta\tau)\pi.G(\pi) \tag{9}$$

and the optimal command is given by

$$(OC) \quad v = V^*(\pi, R) \stackrel{def}{=} A\left(\overline{R}.\pi + \frac{\pi.\overline{R}.\pi}{|\pi|^2}\,\pi \right) \tag{10}$$

We can now formulate the Hamilton-Jacobi equation which yields the solution of (7) $\forall w \in \mathcal{W}$:

$$(HJ^w) \quad \left| \begin{array}{ll} \partial_\tau U^w(\tau, \pi) + H_0\left(\pi, \partial_\pi U^w(\tau, \pi)\right) = 0 & (11.a) \\ U^w(\tau, \pi)_{|\tau=t_0+T} = 0 & (11.b) \end{array} \right. \tag{11}$$

The corresponding optimal command results:

$$(OC^w) \qquad v^w(\tau, \pi) = V^*\left(\pi, \partial_\pi U^w(\tau, \pi)\right) \tag{12}$$

Traveler density

The velocity field v modifies the distribution of probabilities π. In order to keep track at a macroscopic level of this effect, let us introduce the density distribution $\Phi^w(\pi)d\pi$ of probabilities π. This density plays the role of the mean field in the assignment problem. Note that one such density must be defined for each OD $w \in \mathcal{W}$. The dynamics of Φ result by the following conservation equation $\forall w \in \mathcal{W}$:

$$(FP^w) \quad \left| \begin{array}{ll} \partial_\tau \Phi^w(\tau, \pi) + \mathrm{div}\left(\Phi^w(\tau, \pi)\pi\right) = 0 & (13.a) \\ \Phi^w(\tau, \pi)_{|\tau=t_0} = \Phi_{init}(\tau) & (13.b) \end{array} \right. \tag{13}$$

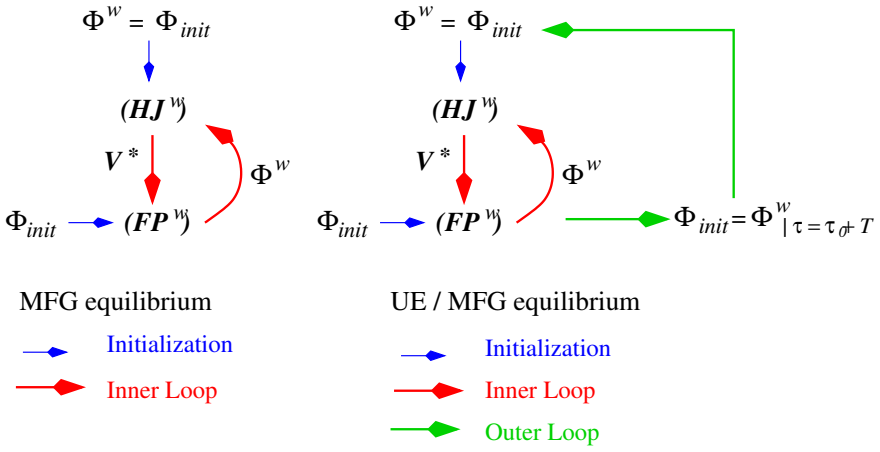


Figure 1: Calculation loop for the solution of the MFG (left) and the user equilibrium via MFG (right). The inner loop must be effectuated for each $w \in \mathcal{W}$. The user equilibrium requires an outer loop.

Costs

At the mean field game limit the number of travelers is considered very large (but the total flows $\sum_{q\in\mathcal{P}_w}\sum_{x\in\mathcal{X}} \mathbb{I}\!\!I^x = D_w$) and the variable $\mathbb{I}\!\!I^x$ admits as a limit a variable $\mathbb{I}\!\!I(\pi)$ such that $\Phi^w \mathbb{I}\!\!I(\pi) d\pi$ is gaussian with mean $\Phi^w(\pi)\pi d\pi$ and variance $\Phi^w(\pi)\mathbf{V}(\pi)d\pi$ (that is the sum of the means and the variances of the choices $\mathbb{I}\!\!I^x$ of the travelers belonging to the set $\Phi^w(\pi)d\pi$). Thus the OD flows $f^w \overset{def}{=} (f_q)_{q\in\mathcal{P}_w}$ are distributed following a gaussian distribution $\mathcal{N}\left(\int \Phi^w(\pi)\pi d\pi, \Phi^w(\pi)\mathbf{V}(\pi)d\pi\right)$. Note that the variance of the sum $\sum_{q\in\mathcal{P}_w} f_q$ is nul, as this sum is equal to D_w.
The path costs depend on all flows $f_q, q \in \mathcal{P}$. We assume that their perception by travelers is the cost of mean flows, thus the contribution of the path costs to the trajectory cost is given by

$$(TC^w) \quad \left| \begin{array}{ll} \pi.G(\pi) = \sum_{p\in\mathcal{P}_w} \pi_p G_q(f) \quad \forall w \in \mathcal{W} & (14.a) \\ f^w = \displaystyle\int \Phi^w(\pi)\pi \, d\pi \quad \forall w \in \mathcal{W} & (14.b) \end{array} \right. \tag{14}$$

Mean field game formulation.

The mean-field formulation of the day-to-day assignment requires the simultaneous resolution of the hamilton Jacobi equation (11) which yields the optimal command by (12), and the conservation equation (13) which yields the trajectory costs by (14) and the reduced hamiltonian (9).
Figure 1 illustrates the procedure. The left side shows the standard MFG loop which moves through steps $(HJ^w), (OC^w), (FP^w)$ for all $w \in \mathcal{W}$ (equations (11), (12) and (13) respectively). This loop results in optimal trajectories of travelers in the space of probabilities π at equilibrium with the impact of the density Φ on travel costs. The right side of Figure 1 shows a process designed to improve the MFG equilibrium by taking as an initial state for the MFG equilibrium the final state of the previous MFG equilibrium. The process results in a outer loop, the inner loop remaining the MFG loop.

3 Results and discussion

The above scheme was tested on a multimodal network of moderate size (two ODs, with respectively 5 and 10 paths joining the ODs) and congested conditions. Convergence results for the outer loop are shown in Figure 2. The convergence is measured by the following UE criterion

$$UE \overset{def}{=} \sum_{w\in\mathcal{W}} \sum_{p\in\mathcal{P}_w} \frac{f_p}{D_w} \left(G_p - \min_{\varpi\in\mathcal{P}_w} G_\varpi \right) \tag{15}$$

UE should go down to 0 at user equilibrium by (4). As expected the convergence is not completely regular since the MFG in the process optimizes both the evolution of probabilities and the expected travel cost.

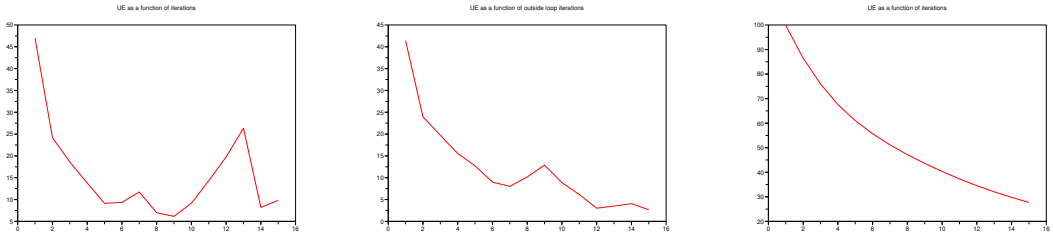


Figure 2: Some examples of convergence of the UE criterion calculated for outer loop iterations. Vertical axis: UE, horizontal axis iteration index. An exponential smoothing was applied from one iteration to the next. The quality of convergence depends on congestion in the network, and structure of demand.

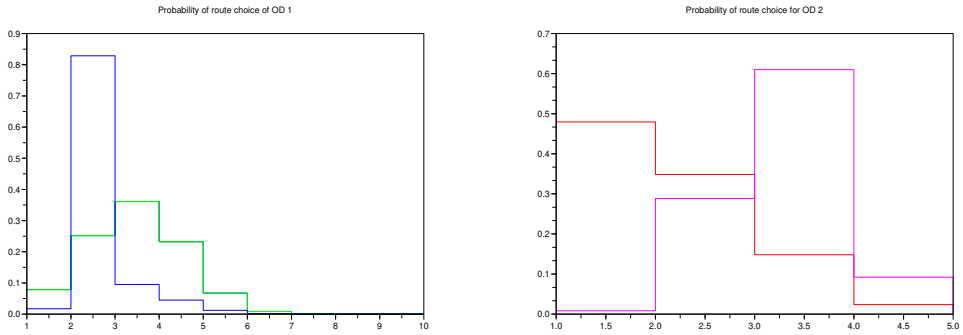


Figure 3: Initial and final route choice probabilities π_q for OD 1 (left) and OD 2 (right). Route indices are carried by horizontal axes, and probabilities by vertical axes. The color codes are the following: left initial probability is in green, the final one in blue, and right the initial probability is in magenta and the final one in red.

Figure 3 shows the path choice probabilities, more precisely the ratios f_p/D_w for the two ODs w and the paths p joining these ODs. The probabilities are shown at the beginning and at the end of the outer loop of Figure 1. The initial probabilities were generated by a logit assignment based on free flow costs in the network. The choice probabilities concentrate on the few more favorable paths. The figure was generated by the middle configuration of Figure 2.

4 Conclusions

Much needs to be done to evaluate the proposed method: improving the convergence and efficiency of the schemes, checking the impact of network size and complexity, checking the impact of congestion costs and capacity bounds. At a more fundamental level, the approach proposed in this paper is comparable, but not identical to the classical utility based approaches, since the main behavioral feature is constituted by the progressive adjustment of individual choice probabilities. It remains to be evaluated to which extent and in which way the results obtained by the approaches differ.

Acknowledgements

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