Optimizing Geometry and Frequency of Transit Networks in Heterogeneous Territories

P. Arcas*[1], H. Badia[2], M. Estrada[3]

[1] PhD candidate, Civil and Environmental Department, Universitat Politècnica de Catalunya – Barcelona Tech, Spain

[2] Associate Professor – Serra Hunter Fellow, Civil and Environmental Department, Universitat Politècnica de Catalunya – Barcelona Tech, Spain

[3] Professor, Civil and Environmental Department, Universitat Politècnica de Catalunya – Barcelona Tech, Spain

SHORT SUMMARY

Traditional transit network design forces a choice between the structural rigidity of discrete graphs and the oversimplified approach of density-based continuous models. This paper bridges that divide by introducing a novel, corridor-based continuum framework that endogenously optimizes spatial coordinates and line-specific headways of transit networks. Driven by a stochastic routing model and a three-stage hybrid optimization strategy, the framework navigates a highly non-convex design space while enforcing vehicle capacity at critical transfer nodes.

Our findings highlight a consistent design pattern: while homogeneous layouts provide good strategic guidance, introducing geometric and frequency heterogeneity balances trade-offs between capacity utilization and accessibility across the network under spatially heterogeneous demand. This fine-tuning improves the efficiency of service allocation, enabling operators to reduce costs by over 13% under centralized heterogeneous demand scenarios, resulting in substantial operational efficiency without degrading the passenger experience.

Keywords: Continuum approximation model, spatial heterogeneity, operations research applications, transport network modelling, transit network design.

1. INTRODUCTION

In urban environments with high population and activity densities, public transport is the most efficient and sustainable mode of mobility. Ensuring its competitiveness requires aligning supply with demand through the rigorous solution of the Transit Network Design and Frequency Setting Problem (TNDFSP) (Ibarra-Rojas et al., 2015). This strategic and tactical planning challenge balances the trade-off of minimizing the resources invested by the operator and maximizing the level of service provided to users.

Historically, the TNDFSP has been addressed through two main modelling paradigms. Discrete formulations represent the network as a graph, optimizing route choices and frequencies over a set of candidate lines and stops. While these models are well suited for incorporating detailed operational constraints in medium to large-scale networks (Schöbel, 2012; López-Ramos, 2014), they typically face severe combinatorial complexity. Their outcomes depend strongly on predefined candidate line pools and network discretization, which often leads to a "black box"

resolution that limits the interpretability of the structural properties of optimal networks (Magnanti and Wong, 1984; Mahmoudi et al., 2024). Conversely, continuum-approximation (CA) models represent demand and service in an aggregated spatial form (Daganzo, 2010; Badia et al., 2014). While CA models offer high analytical tractability and broad strategic insight, they frequently rely on idealized geometries or aggregate density-based representations. These simplifications prevent the explicit optimization of discrete corridor locations and line-specific frequencies, and therefore limit the model's ability to represent the heterogeneous and anisotropic structure of bus operations and their interaction with the territory.

Recent research has sought to adapt the problem to heterogeneous demand patterns by combining continuous and discrete modelling (Chen et al., 2018; Mahmoudi et al., 2025). However, these approaches typically rely on predefined or specifically tailored network layouts, which limit their ability to endogenously determine optimal structures under general heterogeneous demand patterns. Unlike existing hybrid approaches, the proposed framework simultaneously endogenizes geometry, frequency, and capacity constraints within a continuous design space.

To overcome these limitations, this paper proposes a novel corridor-based CA framework that bridges these two perspectives in a feasibility-oriented framework. The city is structured through a set of straight, bidirectional trunk corridors whose spatial coordinates and specific service headways are optimized jointly as continuous variables. The model incorporates a door-to-door routing model that determines passenger paths consistently with the network configuration and allows to explicitly account for vehicle capacity constraints. This approach preserves the strategic interpretability of continuum models while producing discrete, interpretable, and operationally feasible layouts.

The remainder of the paper is organized as follows. Section 2 introduces the proposed corridor-based continuum framework, including the formulation of the optimization problem, the assignment model, and the hybrid solution approach. Section 3 presents the numerical application and discusses the resulting network structures, cost decompositions, and the physical mechanisms underlying the observed efficiency gains. Section 4 concludes with the main insights and directions for future research.

2. METHODOLOGY

The proposed city layout is a square city with side L. Travel demand Λ is distributed over the city with a continuous function $\lambda(i,j)$, being i and j the set of possible origin and destination points.

Transit is structured as an orthogonal network composed of straight, bidirectional bus corridors notated with indices n and m for vertical and horizontal corridors respectively. Such corridors are located in positions x_n and y_m, and served with headways H_n and H_m. Unlike traditional CA models that rely on aggregate route densities, this model endogenously determines the continuous spatial coordinates and the operational headways of each individual corridor alongside the discrete number of lines per direction. This hybrid structure retains the computational efficiency of continuous models while allowing flexible adaptation to spatial demand heterogeneity.

The minimization problem is stated in Equation 1, where the system total cost (Z) is the objective function, defined as the sum of operation (Z_o) and users perceived (Z_u) costs.

$$\min_{N_x,N_y,y_m,x_n,H_m,H_n} Z = Z_o + Z_u \qquad (1.1)$$

$$s.t.$$

$$N_x, N_y \geq 0 \quad (1.2)$$
$$x_n, y_m \in [0, L] \quad (1.3)$$
$$H_n, H_n \geq H_{min} \quad (1.4)$$
$$O_n, O_m \leq C \quad (1.5)$$

Operator costs are captured through the fleet size and total vehicle-kilometers, reflecting the main capital and operating expenditures. Fixed infrastructure costs are excluded under the assumption that the transit service utilizes existing urban street networks. User perceived costs are monetized through a homogeneous value of time μ and includes access, waiting, transfer, and in-vehicle travel times.

Constraint 1.2 ensures the non-negativity of the number of corridors. Constraint 1.3 imposes corridors to be located within the analyzed territory. Constraint 1.4 imposes a minimum headway, avoiding a domain of solutions that are both operationally complex and prone to bunching phenomena, typically considered as unfeasible.

Finally, Constraint 1.5 enforces vehicle capacity limits checked at transfer points. The maximum vehicle occupancy of a bus that serves corridor n (O_n) is obtained evaluating the occupancy of that bus at each intersection with another corridor m, in a conservative manner that considers both boarding and alighting passengers. In this way, the model ensures that passenger boarding remains feasible at the network's most critical nodes. This targeted approach is rigorous enough to satisfy minimum service standards while avoiding the prohibitive computational costs associated with continuous cross-sectional flow aggregations in iterative cost evaluations.

Network connectivity is restricted to direct or one-transfer itineraries, which maintains behavioral realism since this limit captures the vast majority of urban travel patterns given the high perceived disutility of multiple transfers. User behavior is governed by the minimization of disutility through a multinomial logit choice model based on the generalized total travel time associated with each available itinerary.

The resulting joint optimization of corridor geometry and service frequency allocation creates a highly non-convex objective function that is computationally expensive to evaluate. The determination of the number of corridors is intrinsically coupled with frequency allocation decisions, and adjusting the number of lines requires a coordinated relocation of the entire network. If geometry and frequencies are optimized simultaneously from the start, compensatory effects arise. For example, the model may assign artificially high frequencies to poorly positioned corridors to attract demand. This traps the algorithm in suboptimal local minima.

To address these structural complexities, a multi-stage hybrid optimization framework is adopted, transitioning smoothly from global structural exploration to fine-grained continuous refinement. The approach relies on a progressive enrichment of the decision space executed in three sequential steps (see Figure 1).

First, a genetic algorithm establishes a homogeneous baseline by determining the optimal number of corridors and a uniform service area layout, while keeping headways strictly uniform to isolate geometric effects. Second, with the discrete number of corridors fixed, a Sequential Quadratic Programming (SQP) algorithm refines the spatial coordinates of the lines to create a heterogeneous layout, allowing corridors to shift freely according to demand concentration. Finally, a subsequent SQP stage unlocks both free geometry and headway, independently fine-tuning the precise spatial placement and operational frequencies of every individual line. This formal sequence

ensures that resource allocation perfectly aligns with the underlying spatial demand without suffering from premature convergence.

In essence, the optimization proceeds sequentially: it first determines the network structure, then refines corridor geometry, and finally adjusts service frequencies.

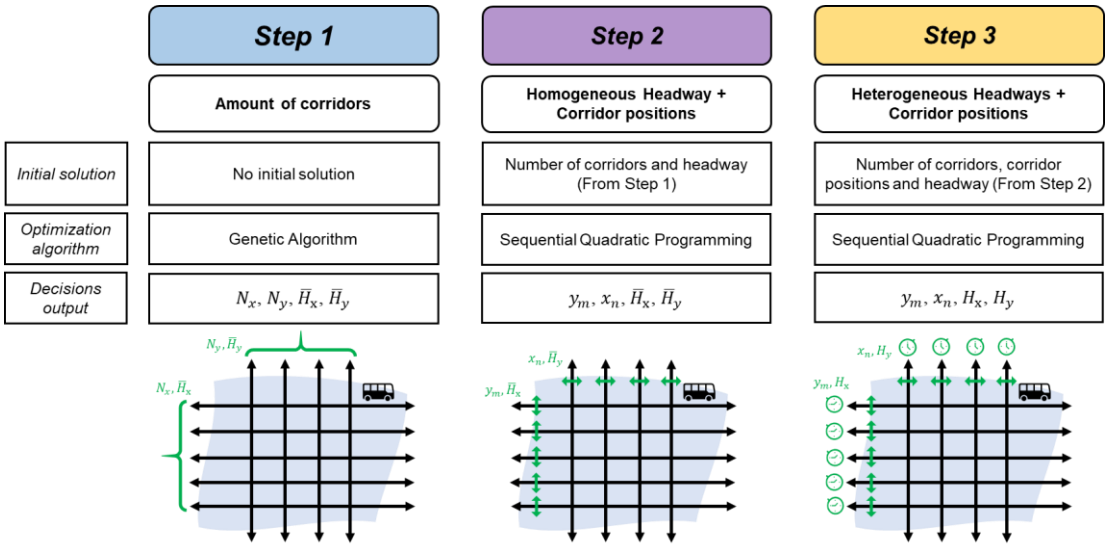


Figure 1. Multistep optimization framework outline.

3. RESULTS AND DISCUSSION

The framework is applied to a stylized square city (10×10 km) with a total fixed demand of $\Lambda = 20{,}000\ pax/h$ and a minimum headway $H_{min} = 2\ min$. To validate the model's sensitivity to demand spatial structure, three patterns are analyzed. First, a homogeneous demand case (C0), with a uniform distribution of origins and destinations. Second, a base case (C1) with a quasi-homogeneous distribution with a central CBD that locally increases demand generation and attraction. Third, a reinforced CBD case (C3) with strong monocentric demand pattern.

By maintaining a constant total demand across scenarios, the experiments allow for a direct comparison of network structures, cost decompositions and the specific impact of heterogeneous design levers. Figure 2 illustrates the solution network configurations in superposition with demand distribution along the city.

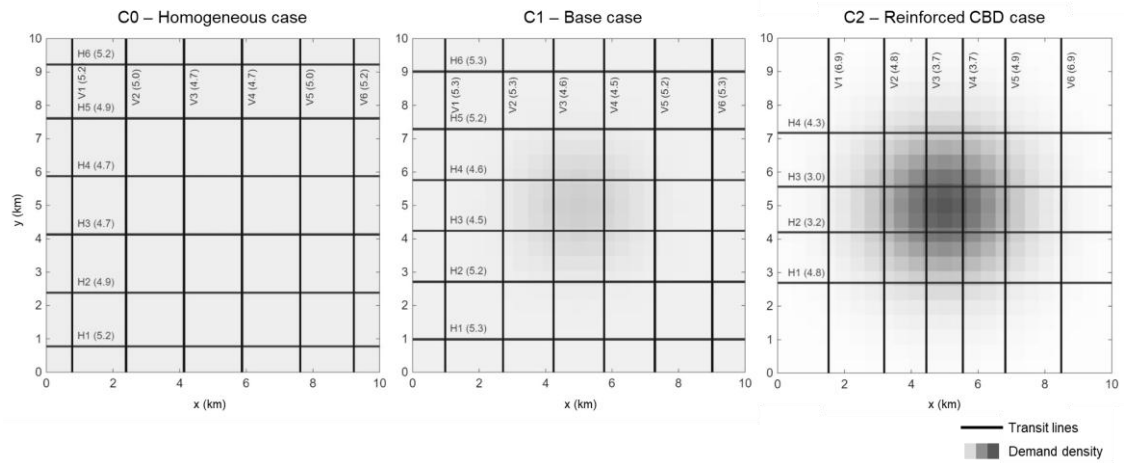


Figure 2. Solution network configurations for experimental cases.

When aggregately analyzing the economic outputs, the staged optimization shows that well-dimensioned homogeneous corridor networks (Step 1) capture most of the system's social welfare. As detailed in Figure 3, across the three demand scenarios, the system total cost (Z) improvements achieved by unlocking heterogeneity remain remarkably limited, with differences below 1% between the rigid baseline and the fully flexible network.

However, the progressive deployment of heterogeneous design levers reveals a clear rebalancing of cost components along Steps 2 and 3. The evolution of users perceived costs (Z_u) highlights that heterogeneous designs achieve substantial operational savings at the expense of only marginal increases in perceived travel costs.

Specifically, in the transition from the homogeneous baseline to the fully free geometry and frequency layout, operator costs decrease by 10.02% in C1 and to 13.04% in C2. These considerable operational savings are achieved with negligible increases in user perceived costs, which rise by a mere 0.87% and 0.58%, respectively. Therefore, heterogeneous designs serve primarily as a powerful tool for operational efficiency, allowing operators to drastically reduce fleet requirements and total vehicle-kilometers without meaningfully degrading the quality of service provided to users. This also indicates that heterogeneity becomes economically relevant primarily under strong spatial demand concentration.

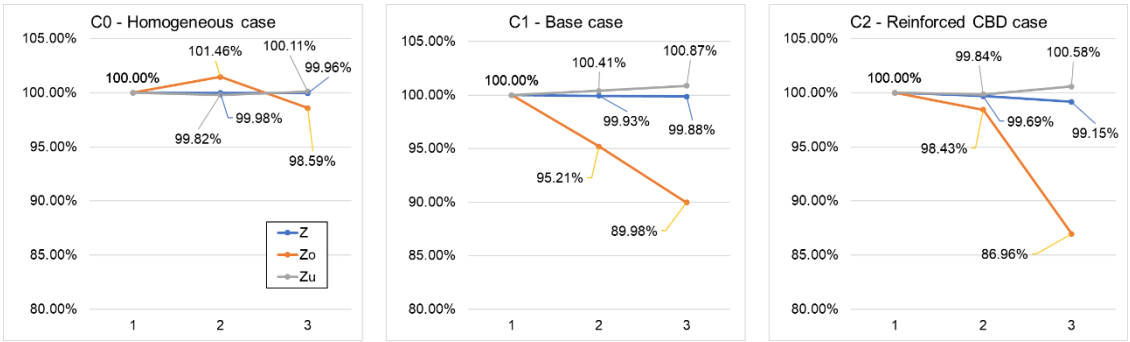


Figure 3. Cost structure variations along multistep optimization framework.

This efficiency is associated with the framework's ability to adapt both network geometry and service allocation to the demand pattern. Figure 2 illustrates these adjustments. In C0, the optimal solution naturally yields a symmetric grid composed of six corridors per direction. Yet, even in

this uniform setting, the model introduces certain heterogeneity: lower headways are assigned to central corridors, with approximately 4.7 minutes, compared to external corridors operating at 5.2 minutes, to adequately process the aggregation of transfer-involving trips in the network's core. As demand centralizes in C1 and C2, Figure 2 shows a concentration of supply around high-demand zones (shorter corridor spacings and lower headways), resulting in lower service provision in peripheral areas.

Vehicle capacity emerges as a key design constraint. Figure 4 shows, for C2, the occupancy profiles of three representative vertical corridors: one peripheral (n = 1), one intermediate (n = 2), and one central (n = 3) from a subset of six corridors. It illustrates, for C2, how occupancy profiles evolve from the homogeneous to the heterogeneous design.

Frequency adjustments reduce excess supply, with passenger occupancies increasing in less-loaded corridors (n = 1) while pushing occupancy close to capacity in central corridors (n = 3) but also in intermediate ones (n = 2). The model internalizes the trade-off between service quality and operating cost, leading to solutions with slightly higher occupancies and overall higher headways, and therefore lower fleet requirements. As a result, the system accommodates demand with fewer vehicles, which is consistent with the observed reduction in operator cost.

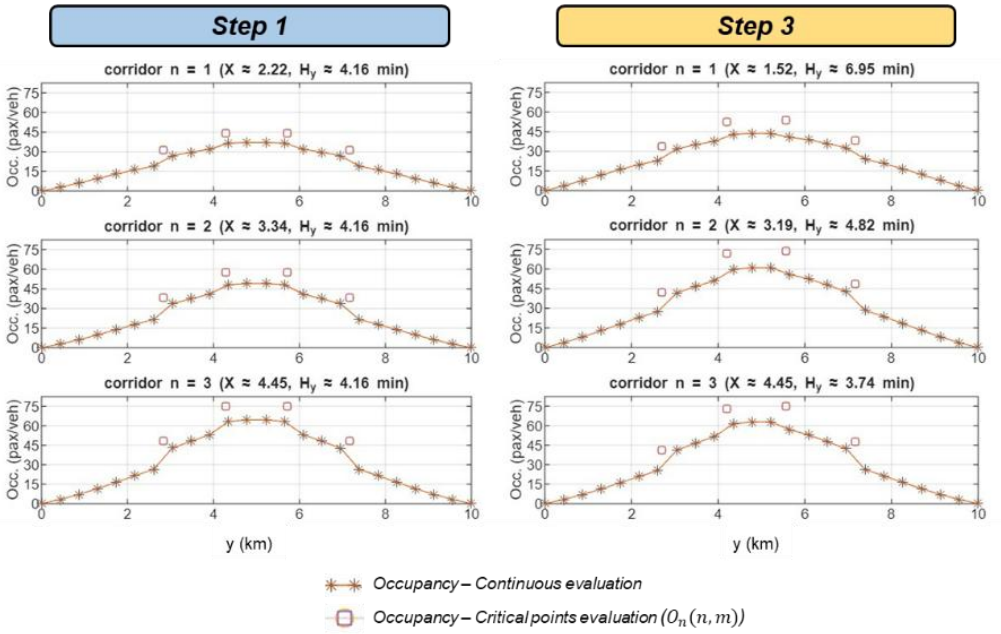


Figure 4. C2. Step 1 to Step 3 occupancy curves of representative vertical lines.

4. CONCLUSIONS

This contribution approaches the TNDFSP within a novel corridor-based continuum-approximation framework that explicitly optimizes continuous spatial geometry and line-specific headways endogenously. By circumventing the traditional reliance on aggregate route densities and predefined candidate graphs, the proposed model successfully bridges the gap between the structural precision of discrete programming and the analytical tractability of continuous models for orthogonal transit networks.

The application of a three-stage hybrid optimization strategy to various urban demand configurations yields three core insights. First, well-dimensioned homogeneous networks provide a robust welfare baseline, capturing the vast majority of social utility regardless of spatial demand variations. Second, heterogeneous network designs act primarily as instruments for operator cost-efficiency; under centralized demand structures, spatial and frequency heterogeneity reduces operating costs by over 13% while keeping user perceived costs with only marginal changes unchanged. Third, these operational savings are strongly associated with the presence of binding vehicle capacity constraints.

The results show that heterogeneous designs lead to different occupancy patterns across corridors, and adapt service supply accordingly. Model outputs maintain territorial coverage while sustaining low occupancies in corridors where service is primarily justified by accessibility considerations. This spatial and frequency adjustment allows the system to accommodate the same demand with a smaller fleet, while internalizing complex and geographically-sensitive trade-offs.

Ultimately, this research provides strategic transit planners with a highly interpretable and operationally relevant tool to evaluate when complex, heterogeneous transit layouts are economically justified. Results suggest that the benefits of introducing heterogeneity in transit network design arise primarily through improved capacity utilization rather than substantial changes in user welfare. Future research will focus on extending this framework to irregular, real-world urban topographies and exploring the integration of multi-modal transit hierarchies.

ACKNOWLEDGEMENTS

The authors acknowledge the financial support of the Spanish Ministry of Science, Innovation and Universities through an FPU grant (FPU24/03868) and the project with reference PID2023-151397OA-100.

REFERENCES

Badia, H., Estrada, M., Robusté, F. [2014]. Competitive transit network design in cities with radial street patterns. *Transportation Research Part B: Methodological*, Vol. 59, pp. 161-181.

Chen, J., Liu, Z., Wang, S., & Chen, X. [2018]. Continuum approximation modeling of transit network design considering local route service and short-turn strategy. *Transportation Research Part E: Logistics and Transportation Review*, Vol. 119, pp. 165-188.

Daganzo, C. F. [2010]. Structure of competitive transit networks. *Transportation Research Part B: Methodological*, Vol. 44, No. 4, pp. 434-446.

Guihaire, V., & Hao, J. K. [2008]. Transit network design and scheduling: A global review. *Transportation Research Part A: Policy and Practice*, Vol. 42, No. 10, pp. 1251-1273.

Ibarra-Rojas, O. J., Delgado, F., Giesen, R., & Muñoz, J. C. [2015]. Planning, operation, and control of bus transport systems: A literature review. *Transportation Research Part B: Methodological*, Vol. 77, pp. 38-75.

López-Ramos, F. [2014]. Integrating network design and frequency setting in public transportation networks: A survey. *Statistics & Operations Research Transactions,* Vol. 38, No. 2, pp. 181-214.

Magnanti, T. L., & Wong, R. T. [1984]. Network design and transportation planning: Models and algorithms. *Transportation Science*, Vol. 18, No. 1, pp. 1-55.

Mahmoudi, R., Saidi, S., Wirasinghe, S. C. [2024]. A critical review of analytical approaches in public bus transit network design and operations planning with focus on emerging technologies and sustainability. *Journal of Public Transportation*, Vol. 26.

Mahmoudi, R., Saidi, S., Wirasinghe S.C. [2025]. Expansion of bi-modal express transit networks - a hybrid optimization approach, *Transportation Research Part E: Logistics and Transportation Review*, Vol. 202.

Schöbel, A. [2012]. Line planning in public transportation: Models and methods. *OR Spectrum*, Vol. 34, No. 3, pp. 491-510.