New Formulations for the Capacitated Choice-Based Line Planning Problem

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Short summary

In this paper, we integrate route choice models into capacitated line planning with endogenous demand. The resulting problem is formulated as a bilevel mixed-integer program, where route choices are modeled using sampling-based techniques, yielding a flexible but computationally challenging framework. We propose two single-level reformulations: an arc-based formulation relying on strong duality, and a path-based formulation. Surprisingly, numerical experiments indicate that, under specific assumptions, the path-based formulation consistently outperforms the arc-based one.

Keywords: bilevel optimization, discrete choice models, line planning problem, network design, user equilibrium.

1 Introduction

In response to growing awareness of the climate crisis, improving the efficiency and level of service of public transportation networks (PTNs) has become a key priority in urban planning. For instance, the modal shift from individual vehicles to public transportation is more likely to occur when the network is sufficiently connected. Therefore, the line planning problem (LPP), aiming to design a PTN under budget and operational constraints, plays a central role in improving system performance and attracting demand.
The behavioral responses to PTNs can be modeled using advanced discrete choice models (DCMs), which capture individual preferences at a disaggregated level. However, accounting for these behavioral heterogeneity is challenging, as individual choices are inherently stochastic and the resulting choice probabilities often lack closed-form expressions. Moreover, the passengers are assumed to be selfish agents, leading to user equilibrium (UE): each passenger aims at minimizing their own utility function (e.g., travel time). Since no compromise occurs between passengers, the solution may suboptimal compared to the system optimum (SO), obtained by assigning passengers to routes.

Literature Review

The LPP has been extensively studied as a core component of public transportation network design (see, e.g., Schöbel (2012); Owais (2026)). Although many variations of the problem exist, they typically consist in selecting a set of transit lines and their operating frequencies from a predefined line pool, given an underlying network of stations and a demand pattern. In LPP, and more generally in network design, many works focus on minimizing the system operation costs, accounting for passenger satisfaction either with outsourcing penalty, as in Hartleb et al. (2021), or through demand-dependent benefits, as in Nicolet & Atasoy (2024). However, the LPP is particularly relevant in the context of public planning, where transport authorities aim to maximize the level of service or passenger satisfaction under budget constraints, as discussed by Schöbel (2012) and Goerigk & Schmidt (2017). In most formulations, passenger routing is assumed to be known a priori or described by fixed probability distributions. As a result, demand is treated as exogenous, neglecting the interaction between network design and user behavior, as in Schöbel (2012) and Kepaptsoglou & Karlaftis (2009).

To address this limitation, a growing number of works integrate demand modeling into network design problems. One approach consists in refining demand prediction while keeping it exogenous, for instance through data-driven or predictive demand models, following the approach of Bertsimas et al. (2021). When UE is assumed, demand and routing may depend on choice- or design-dependent parameters such as congestion or level of service, leading to an endogenous modeling of demand, as shown by Wei et al. (2022) and Etebari (2019). More refined demand representations rely on logit models, a simple class of DCMs in which choice probabilities admit closed-form expressions, although they remain highly nonlinear (see, e.g., Hartleb et al. (2021)). However, most existing approaches still rely on simplified or aggregated demand representations, limiting their ability to capture individual behavioral responses.
In transportation, DCMs are widely used to represent passenger routing and assignment decisions, capturing how individuals select routes based on attributes such as travel time and cost (see, e.g., Frejinger et al. (2009); Ricard & Bierlaire (2025)). The integration of DCMs into optimization problems enables demand to respond endogenously to design decisions, but significantly increases computational complexity, as shown by Nicolet & Atasoy (2024), Etebari (2019) and Paneque et al. (2019). Following Train (2009), simulation-based methods such as Monte Carlo sampling allow the use of more flexible DCMs beyond simple logit models, particularly when choice probabilities do not admit closed-form expressions. Nevertheless, it adds to the computational complexity, as it requires repeated resolutions. Consequently, combining behavioral realism with tractable optimization remains a major challenge.
Goerigk & Schmidt (2017) and Rey & Levin (2024) proposed UE formulations, resulting in bilevel problems. Indeed, passengers are independent agents who each maximize their own utility, which depends on the operator's decisions. In this bilevel setting, the leader is the system operator solving a LPP; and the followers are the passenger making route choices. In the absence of capacity constraints, the objectives of the operator and the passengers are naturally aligned, which can lead to equivalent single-level formulations. However, when capacity constraints are present and under a SO formulation, the operator may assign passengers to suboptimal routes to improve global utility. While some LPP settings allow the use of strong duality to obtain single-level reformulations, following Goerigk & Schmidt (2017), alternative approaches instead relax the UE assumption through SO-based branch-and-bound algorithms developed by Rey & Levin (2024).

Contributions

Most previous work in the line planning literature avoids computational challenges by simplifying the modeling of demand or relying on heuristic solution methods. Some approaches assume fixed route assignments, sometimes incorporating stochasticity through aggregated closed-form probabilities. Other works adopt a SO perspective, in which passengers are assigned to routes by the operator, thereby keeping the problem at a single level. More recent studies have started to address the exact solution of the UE line planning problem.
Overall, existing approaches address only subsets of the computational challenges. In particular, no existing work simultaneously accounts for (i) endogenous demand based on disaggregate behavioral models, (ii) non-closed-form choice probabilities, and (iii) the bilevel structure induced by user equilibrium. This gap motivates the development of new exact methods for the capacitated choice-based line planning problem (CCB-LPP). In this work, we propose a new formulation of the CCB-LPP, in which endogenous demand is modeled using a DCM. By relying on Monte Carlo sampling, the framework accommodates a wide range of choice models, without requiring closed-form choice probabilities or leading to complex nonconvex optimization problems. We further develop two single-level reformulations and experimentally compare their computational performance. To the best of our knowledge, this is the first work to exactly solve a choice-based LPP while allowing for advanced DCMs to represent demand.

2 Methodology

Bilevel line planning problem

We consider a capacitated LPP in which the network is represented by a spatial graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V}$ denotes the set of stations and $\mathcal{A}$ the set of physical connections. Demand is given by an origin–destination (OD) matrix, which specifies the volume of passengers traveling between each pair of stations. The operator selects a subset of transit lines from a predefined line pool $\mathcal{L}$, where

each line $\ell \in \mathcal{L}$ corresponds to a path, i.e. an ordered sequence of visited stations in the network representing a feasible bus route. We denote by $N_{\mathcal{V}}$ the number of stations and $N_{\mathcal{L}}$ the number of lines in the pool. For each line $\ell \in \mathcal{L}$, the operator first decides whether to open the line through the binary variable y_ℓ, and, if so, determines its operating frequency $f_\ell \in 1, \dots, F_\ell$, representing the number of vehicles scheduled per hour. Opening a line ℓ incurs a fixed cost C_ℓ^F as well as an operational cost assumed to be linear in the line frequency, i.e., $C_\ell^O f_\ell$. The total cost of a feasible line plan $(y, f) \in \{0,1\}^{|\mathcal{L}|} \times [1, \dots, F_\ell]^{N_\mathcal{L}}$ must not exceed a given budget B. We denote by x the route decision of the passengers. It can be represented either by arc-based variables x_{rn}^a, equal to 1 if arc $a \in \mathcal{A}$ is used, or by path-based variables x_n^p, equal to 1 if path $p \in \mathcal{P}_n$ is chosen, where $\mathcal{P}_n$ denotes the set of feasible paths for passenger n. Likewise, u_n^a (resp. u_n^p) is the utility associated with arc $a\mathcal{A}$ (resp. with path $p \in \mathcal{P}_n$). The CCB-LPP at hands writes:

$$\max_{y,f} \quad \sum_{n \in N} \sum_{p \in \mathcal{P}_n} u_n^p x_n^p \left(x_n\right) \tag{1}$$

$$\text{s.t.} \quad \sum_{\ell \in \mathcal{L}} \left(C_\ell^O f_\ell + C_\ell^F y_\ell\right) \leq B \tag{2}$$

$$f_\ell \leq F_\ell \qquad \forall \ell \in \mathcal{L} \tag{3}$$

$$s_n^a \leq \sum_{\ell \in \mathcal{L}} H_a^\ell Q_\ell f_\ell - \sum_{m=1}^{n-1} x_m^a \qquad \forall n \in N, a \in \mathcal{A} \tag{4}$$

$$x_n \textbf{ is optimal for } (FP_n(f, y, s)) \qquad \forall n \in N \tag{5}$$

$$s_n^a \in \mathbb{N} \qquad \forall n \in N, a \in \mathcal{A}$$

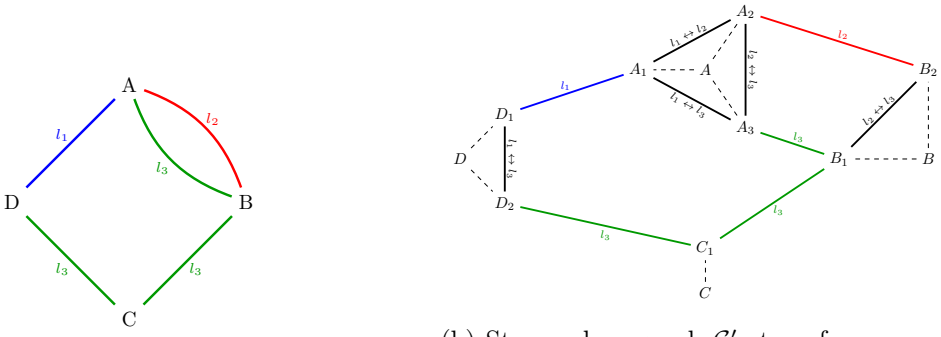
$$y_\ell \in \{0,1\}, f_\ell \in \mathbb{N} \qquad \forall \ell \in \mathcal{L},$$

where H_a^l is an indicator parameter that takes value 1 if line $\ell \in \mathcal{L}$ goes through arc $a \in \mathcal{A}$, and Q_ℓ is the vehicle capacity of line ℓ.
The system operator aims to maximize overall utility (1) subject to budget constraint (2) and bounded line frequencies (3). To simulate the allocation of capacity across passengers, we rely on an exogenous priority queue, as in Binder et al. (2017): after sorting passengers in a random order, arc capacities are progressively filled according to this order. Therefore, when passenger n chooses a route, any line segment whose capacity $Q_\ell f_\ell$ has already been fully used by previously assigned passengers $m < n$ is no longer available. Constraints (4) define variables s, where s_n^a captures the remaining capacity on arc a for passenger n. Variables y, f and s are inputs to each follower problem $(FP)_n$, which models the route choice of passenger n. These assignments are simulated using Monte Carlo sampling on a DCM and solved as a mixed-integer (non) linear program (MI(N)LP) and are further detailed next.

Passenger route choice

Passengers choose the route that maximizes their utility, as defined in DCM theory. The utility function u_n^p reflects the attractiveness of path p for individual n, consisting of a deterministic component V_n^p (e.g., travel time, number of transfers) and a stochastic component. We consider arc-specific stochastic components η_n^a and an alternative-specific component ε_n^p. This setting corresponds, for instance, to mixed logit models commonly used in DCM. Using Monte Carlo sampling, we relax distributional assumptions, allowing for arbitrary DCM specifications as long as they can be sampled. Consequently, the route choice is solved for R independent scenarios, with ε_{rn}^p and η_{rn}^a fixed for each $r \in R$. From this point on, variables and parameters impacted by the sampling carry an additional index $r \in R$.
To capture line-specific movements and transfers, we construct a stop-and-go graph $\mathcal{G}'$ associated with $\mathcal{G}$ and the line pool $\mathcal{L}$. For instance, in the graphs of Figure 1, the path $A - B$ in $\mathcal{G}$ is distinguished into two paths, namely $A - A_2 - B_2 - B$ and $A - A_3 - B_1 - B$ in $\mathcal{G}'$. For the sake of readability, this example is based on a non-oriented graph $\mathcal{G}$, although transfer arcs are asymmetric in our model.
Both in-vehicle travel time t^{iv} and transfer time t^{tr} are additive attributes that are fully captured along any path in the stop-and-go graph. In this section, we will focus on those attributes to specify the DCM. Passenger n's utility for route p is given by:

$$u_{rn}^p = \sum_{a \in p} u_{rn}^a + \varepsilon_{rn}^p = \sum_{a \in p} \left(\beta_{iv} t_a^{iv} + \beta_{tr} t_a^{tr} + \eta_{rn}^a\right) + \varepsilon_{rn}^p \ , \tag{6}$$



(a) Original network $\mathcal{G}$ with line definitions.

(b) Stop-and-go graph $\mathcal{G}'$: transfer arcs are labeled $\ell_i \leftrightarrow \ell_j$; dashed arcs denote entry/exit at stations.

Figure 1: Illustration of the construction of a stop-and-go graph

assuming t_a^{iv} (resp. t_a^{tr}) is null when a is a transfer arc (resp. non-transfer arc), with β parameters calibrated from previously estimated DCMs. The follower problem is now formulated as an MILP:

$$\max_x \quad u_{rn}^p x_{rn}^p \tag{7}$$

$$u_{rn}^p = \sum_{a \in p} u_{rn}^a + \varepsilon_{rn}^p \qquad \forall p \in \mathcal{P}_{rn} \tag{8}$$

$$\sum_{p \in \mathcal{P}_{rn}} x_{rn}^p = 1 \tag{9}$$

$$x_{rn}^p \leq I_a^p s_{rn}^a \qquad \forall a \in \mathcal{A} \tag{10}$$

$$x_{rn}^p \in \{0,1\} \qquad \forall p \in \mathcal{P}_{rn}$$

where $I_a^p \in \{0,1\}$ is an indicator parameter that is equal to 1 if arc a belongs to path p. The formulation is linear when the leader variables are fixed. When f are decision variables, non linearities arise from the assumption that oriented transfer times $t_a^{tr} = t_{\ell_i \to \ell_j}^{tr}$ are inversely proportional to the frequency f_j of the receiving line, i.e. proportional to the headway. The set of feasible paths $\mathcal{P}_{rn}$ for passenger n in scenario r also depends on the open lines y. Constraint (10) accounts for the remaining capacity.
While bilevel formulations can be solved as such using fix-and-optimize and other heuristics (see Colson et al. (2007)), exact resolutions typically imply single-level formulations, introduced below.

Single-level Reformulations

We developed two single-level reformulations of the bilevel CCB-LPP. The strong duality (SD) reformulation, commonly used in the bilevel literature, requires stronger assumptions on the stochastic components of the DCM. The discounted utility (DU) approach relies on a path-based formulation and offers greater flexibility in modeling route choice.
As an approximation of a choice model, alternative-specific stochastic component ε_{rn}^p may be omitted from the choice simulation, implying that most variability is captured by the arc-level components η_{rn}^a. Under this simplifying assumption, the structure of the follower problem (FP_{rn}) can be formulated as an arc-based shortest path problem:

$$\max_x \quad \sum_{a \in \mathcal{A}} u_{rn}^a x_{rn}^a \tag{11}$$

$$\sum_{a^+ \in \delta^+(i)} x_{rn}^{a^+} - \sum_{a^- \in \delta^-(i)} x_{rn}^{a^-} = \theta_i \qquad i \in \mathcal{K} \tag{12}$$

$$x_{rn}^a \leq s_{rn}^a \qquad \forall a \in \mathcal{A} \tag{13}$$

$$x_{rn}^a \in \{0,1\} \qquad \forall a \in \mathcal{A} \tag{14}$$

where $\delta^+(i)$ (resp. $\delta^-(i)$) is the set of arcs entering (resp. leaving) node i, and θ_i is equal to 1 if i is the origin node $O(r,n)$ for passenger n in scenario r, -1 if i is the destination node $D(r,n)$ and 0 otherwise. Since in the follower problem, s_{rn}^a are fixed, this reduces to a shortest path problem in

a subgraph $\bar{\mathcal{G}}'$ of $\mathcal{G}'$, obtained by removing arcs with no remaining capacity. The flow constraints (12) define a totally unimodular constraint matrix, implying that the integrality constraints (14) can be relaxed, as the problem shares its solution with its continuous relaxation. Consequently, a single-level reformulation of our CCB-LPP based on the strong duality of the follower problem can be applied, as in Goerigk & Schmidt (2017). We denote by π^i_{rn}, for $i \in \mathcal{K}$, the dual variables associated with constraints (12), and by α_{rn} the dual variables associated with constraints (13). The resulting formulation is given by:

$$
\begin{aligned}
(ST) \quad \max_{y,f} \quad & \sum_{r\in R}\sum_{n\in N}\sum_{a\in\mathcal{A}} u^a_{rn}x^a_{rn} && && (15)\\
\text{s.t.} \quad & \sum_{\ell\in\mathcal{L}} \left(C^O_\ell f_\ell + C^F_\ell y_\ell\right) \le B && && (2)\\
& f_\ell \le F_\ell && \forall \ell \in \mathcal{L} && (3)\\
& s^a_{rn} \le \sum_{\ell\in\mathcal{L}} H^\ell_a f_\ell - \sum_{m=1}^{n-1}\sum_{p\in\mathcal{P}_{rn}} I^p_a x^p_{rm} && \forall r\in R, n\in N, a\in\mathcal{A} && (4)\\
& \sum_{N,R}\sum_{A} u^a_{nr}x^a_{nr} = \pi^{rn}_{D(r,n)} - \pi^{rn}_{O(r,n)} + \sum_A s^a_{rn}\alpha^a_{rn} && \forall r\in R, n\in N && (16)\\
& x^a_{rn} \le s^a_{rn} && \forall r\in R, n\in N, a\in\mathcal{A} && (17)\\
& \sum_{a^+\in\delta^+(i)} x^{a^+}_{rn} - \sum_{a^-\in\delta^-(i)} x^{a^-}_{rn} = \theta_i && \forall r\in R, n\in N, i\in\mathcal{V} && (18)\\
& \pi^j_{rn} - \pi^i_{rn} + \alpha^{ij}_{rn} \ge u^{ij}_{rn} && \forall r\in R, n\in N, (ij)\in\mathcal{A} && (19)\\
& \alpha, \pi, x \ge 0 && && \\
& y \in \{0,1\}^{N_\mathcal{L}} && && \\
& f, s \in \mathbb{N} && &&
\end{aligned}
$$

Primal and dual objective values of the follower problem must be equal, as enforced by constraint (16). Constraints (17) and (18) are derived from the primal and constraints (19) are derived from the dual of the follower problem.

Lifting the simplifying hypothesis on the stochasticity in the DCM, the ε^p_{rn} terms prevents a fully arc-based formulation, and hence an SD reformulation. In this more general case, an exact single-level formulation can be obtained by explicitly bounding the utility from below so that it takes its maximum value. This DU approach is used in a choice-based optimization context by Haering et al. (2024). Applying this method to the CCB-LPP yields:

$$
\begin{aligned}
(DU) \quad \max_{y,f} \quad & \sum_{r\in R}\sum_{n\in N} u_{rn} && && (20)\\
\text{s.t.} \quad & \sum_{\ell\in\mathcal{L}} \left(C^O_\ell f_\ell + C^F_\ell y_\ell\right) \le B && && (2)\\
& f_\ell \le F_\ell && \forall \ell \in \mathcal{L} && (3)\\
& s^a_{rn} \le \sum_{\ell\in\mathcal{L}} H^\ell_a f_\ell - \sum_{m=1}^{n-1}\sum_{p\in\mathcal{P}_{rn}} I^p_a x^p_{rm} && \forall r\in R, n\in N, a\in\mathcal{A} && (4)\\
& z^p_{rn} \ge 1 - \sum_{a\in p} s^a_{rn} && \forall r\in R, n\in N, p\in\mathcal{P}_{rn} && (21)\\
& u_{rn} \ge u^p_{rn}x^p_{rn} - M z^p_{rn} && \forall r\in R, n\in N, p\in\mathcal{P}_{rn} && (22)\\
& u_{rn} = \sum_{p\in\mathcal{P}_{rn}} u^p_{rn}x^p_{rn} && \forall r\in R, n\in N && (23)\\
& \sum_{p\in\mathcal{P}_{rn}} x^p_{rn} = 1 && \forall r\in R, n\in N && (24)\\
& x^p_{rn}, z^p_{rn} \in \{0,1\} && \forall r\in R, n\in N, p\in\mathcal{P}_{rn} && \\
& s^a_{rn} \in \mathbb{N} && \forall n\in N, a\in\mathcal{A} && \\
& y_\ell \in \{0,1\}, f_\ell \in \mathbb{N} && \forall \ell\in\mathcal{L} &&
\end{aligned}
$$

In addition to the previous constraints, constraints (23) and (24) ensure that u_{rn} takes the value of the utility of exactly one path. Constraints (21) ensure that if an arc capacity is saturated

along a path p for passenger n in scenario r, then variable z^p_{rn} takes value 1. This way, variables z account for the unavailability of alternatives. The maximum utility must then be greater than all discounted utilities $u^p_{rn}x^p_{rn} - Mz^p_{rn}$, as enforced by constraints (22), where M is a constant greater than any non-discounted utility u^p_{nr}.
The main drawback of this DU formulation is the exponential number of constraints (22). Indeed, the maximum utility must be at least as large as the discounted utility of every alternative, which correspond to paths in a strongly connected graph. Contrarily to the DU formulation, in the SD approach, the number of additional constraints grows polynomially with the problem size. However, new nonlinearities arise from the product $s^a_{rn}\alpha^a_{rn}$ of constraints (16). The SD formulation also relies on the approximation that removes the alternative-specific term ε^p_n, whereas the DU formulation remains valid in both cases.
We now assess the performance of the proposed single-level reformulations SD and DU on a set of numerical experiments, focusing on computational efficiency.

3 Results and discussion

In this section, we present an experimental comparison of the scalability of the DU and SD single-level reformulations of the CCB-LPP. Computations are performed on a machine equipped with an Intel(R) Core(TM) Ultra 7 268V processor (2.20 GHz, 8 cores), running Windows and Gurobi 12.0.3. As the primary objective of these experiments is to assess tractability, the DCM specifications (β parameters and distributions of η) are set arbitrarily, as they do not affect computational performance. However, the framework can accommodate most DCM specifications when such models are available from socio-economic studies.
We begin with an analysis of the scalability with respect to instance size in a single-scenario setting. Then, the impact of the number of scenarios on small instances is examined.

Instance generation

Our instance generation follows the approach outlined in Goerigk & Schmidt (2017), as their LPP modeling closely aligns with our CCB-LPP. Given a number of stations $N_{\mathcal{V}}$, we uniformly generate their locations in a 1×1 square. We then uniformly distribute $N = 10N_{\mathcal{V}}$ passengers (or aggregated groups of passengers) origins and destinations across the vertices. To create a realistic graph structure, we apply the k-nearest neighbors algorithm, ensuring that the arcs $\mathcal{A}$ are neither overlapping nor unrealistically long. Using random walks in $\mathcal{G}(\mathcal{V}, \mathcal{A})$, we build $3N_{\mathcal{V}}$ lines of at least 3 stops each, that constitute the line pool. Their cost is proportional to their length with a uniform scaling factor. Once $\mathcal{G}$ and the line pool is built, we create the corresponding stop-and-go graph $\mathcal{G}' = (\mathcal{V}', \mathcal{A}')$ and place the generated demand. In the numerical experiments, the number of transfers is restricted to at most 2, as passengers typically accept only a limited number of transfers. The time limit is set to 3600 seconds.
We first study performance as the instance size $N_{\mathcal{V}}$ increases in a single-scenario setting, then fix small instances and vary the number of scenarios R. For each pair $(R, N_{\mathcal{V}})$, both formulations are tested on five randomly generated instances.

Impact of instance size

We consider single-scenario instances of size $N_{\mathcal{V}}$, with $N_{\mathcal{V}} \in [4, \ldots, 14]$. Table 1 reports the instance specifications, the average computation times, and the number of instances from the set that reach optimality within the time limit (# opt. sol.) for SD and DU. Column $\overline{N}_{\mathcal{V}'}$ and $\overline{N}_{\mathcal{A}'}$ represent the average number of nodes and arcs in the stop-and-go graphs in the instance set.
Table 1 shows that DU formulation outperforms SD formulation for all instances. This dominance is systematic, though relatively modest for small instances. To assess variability, we compute the relative standard deviation (RSD), defined as the ratio of the standard deviation σ of the computation times to their mean $\bar{t}$, i.e., $RSD = \frac{\sigma}{\bar{t}}$. Averaging over all experiments for each method yields $RSD_{DU} = 0.28$ and $RSD_{ST} = 0.43$, indicating that DU formulation is more stable.

Instance					SD		DU	
$N_{\mathcal{V}}$	$\overline{N}_{\mathcal{V}'}$	$\overline{N}_{\mathcal{A}'}$	$N_{\mathcal{L}}$	N	time (s)	# opt. sol.	time (s)	# opt. sol.
4	43.8	283	12	40	71.3	5/5	16.7	5/5
5	55.8	384	15	50	242.7	5/5	28.6	5/5
6	67.4	477	18	60	122.2	5/5	44.3	5/5
7	79.6	570	21	70	224.3	5/5	101.8	5/5
8	90.2	646	24	80	174.1	5/5	118.7	5/5
9	103	744	27	90	402.3	5/5	276.0	5/5
10	111	788	30	100	581.5	5/5	392.8	5/5
11	125	882	33	110	1001	3/5	608.3	5/5
12	136	1000	36	120	991.5	5/5	592.7	5/5
13	146	1046	39	130	1287.3	5/5	761.8	5/5
14	160	1159	42	140	2251.2	4/5	1175.0	5/5

Table 1: Performance comparison between SD and DU formulations when increasing $N_{\mathcal{V}}$

These results should be interpreted with caution due to the imposed limit of two transfers. Without this cap, the number of acceptable paths grows exponentially, which would increase the computational burden, particularly for the DU formulation that introduces one constraint per path.

Impact of sampling size

We now assess the impact of an increase in the number of scenario R. Here again, the DU formulation outperforms the SD formulation. From 15 scenarios onward, the SD approach fails to converge within the time and memory limit for all instances (see Table 2). When increasing the number of scenarios to $R = 25$ and the time limit to 5000 seconds, four out of five instances are solved by the DU formulation within the time limit. In contrast, the SD formulation fails to solve any of the five instances, as it reaches the memory limit (Gurobi status), which highlights the poor scalability of the approach.

Instance	SD		DU	
R	time (s)	# opt. sol.	time (s)	# opt. sol.
1	34.0	5/5	14.3	5/5
5	954.1	5/5	303.0	5/5
10	2493.9	5/5	701.3	5/5
15	3600.0	0/5	1113.4	5/5
20	3600.0	0/5	2739.7	4/5

Table 2: Performance comparison between SD and DU formulations when increasing R

Using the values from Table 2, together with the $R = 25$ experiment with an extended time limit, and excluding instances that did not converge, we plot the average computation times for the SD (red dashed line) and DU (blue line) formulations in Figure 2. This illustrates the nonlinear relationship between sampling and complexity, as the scenario-dependent follower problems are coupled through the leader decisions, even though some constraints, such as capacity, exhibit a block structure.

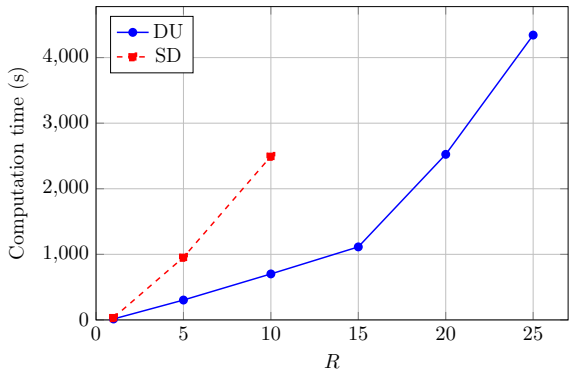


Figure 2: Computation time as a function of the number of scenarios R

4 Conclusions

In public transport network design, the level of service remains a central objective. Discrete choice models provide a powerful framework to capture passenger preferences in a realistic and disaggregate manner. However, such models are rarely integrated into exact optimization approaches for the line planning problem, due to the resulting computational complexity.
In this paper, we incorporated DCM-based route choice into a capacitated LPP with endogenous demand. We proposed two single-level reformulations. The SD approach, commonly used in bilevel programming, is compatible with certain arc-based choice models but cannot accommodate more general specifications such as mixed logit. In contrast, the DU formulation offers greater flexibility and can handle a wider range of DCMs.
Computational experiments show that, under assumptions on the maximum number of transfers, the DU formulation outperforms the SD formulation and exhibits lower variability, while lifting assumptions on the DCM used.
Nevertheless, both approaches face scalability limitations, highlighting the need for further methodological developments. Moreover, both formulations rely on arc-additive utility. For instance, capturing non-additive effects, such as the nonlinear impact of the number of transfers, would require more refined modeling.

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