Optimal planning model for shared charging stations considering spatial heterogeneity and multi-types of chargers: Evidence from Barcelona

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SHORT SUMMARY

To further improve the development of electric vehicles (EVs), this research proposes a model based on continuous approximation (CA) for the optimal planning of shared charging stations. By considering spatial heterogeneity and multi-types of chargers, the planning optimization problem is formulated as a mixed-integer nonlinear optimization problem (MINLP) with objective to minimize the total time spent on charging related activities, which are composed of two parts named travel time and waiting time, under both budget and power supply constraints. A new planning variable, number of fast chargers per station has been incorporated with density and capacity of the charging station. Results from Barcelona case study show strong potential of the proposed planning model on reducing overall total time spent on charging related activities with less investment budget compared with the current layout, highlighting the significance of considering spatial heterogeneity when planning shared charging facilities.

Keywords: Shared charging stations, spatial heterogeneity, charging demand, continuous approximation, fast charger, power supply.

1. INTRODUCTION

Global climate change and energy scarcity have made EVs a key solution for reducing traffic emissions and petroleum dependence. However, the large-scale deployment of EVs poses significant challenges for charging infrastructure planning, particularly for shared charging stations that require long-term and high-cost investments. Efficient planning is therefore essential to alleviate range anxiety and support widespread EV adoption. Existing studies have primarily formulated charging station planning as optimization problems with various objectives, such as minimizing charging time, maximizing operator profit, and reducing total system cost (Lokhandwala & Cai, 2020; Liu et al., 2024; Wang et al., 2023; Zeng et al., 2025). Other works focus on improving utilization and serviceability through stochastic decision processes or CA approaches (Ye et al., 2025; An et al., 2025). Additionally, constraints such as budget limits and power grid capacity have been incorporated to enhance model realism and robustness(Lokhandwala & Cai, 2020; Wang et al., 2023; Liu et al., 2024). Despite these advances, limited efforts can be found when jointly considering spatial heterogeneity and multi-types of chargers in charging station planning. To address this gap, this study proposes a parsimonious CA-based model for shared charging station planning. This model decomposes a complex heterogeneous problem into a set of smaller and homogeneous problems that approximate the local spatial-temporal neighborhood of each problem. While CA method has been widely applied in various domains, such as system design and facility location (Soriguera and Jiménez-Meroño 2020; Jiménez and Soriguera 2024), its application to optimal shared charging station planning under the consideration of budget and power stability simultaneously, as well as the spatial heterogeneity, is a novel endeavor.

2. METHODOLOGY

The proposed CA-based model aims to determine the optimal configuration of shared charging stations under spatial heterogeneity. The objective is to minimize total time spent on charging related activities, subject to budget and power supply constraints. The following sections present the model definition, charging demand modeling and the analytical modeling of time in detail.

Model overview and decision variables

A continuous two-dimensional service region R is considered where all charging demand is generated. The density of charging station is Δ [stations/km^2], which is one of the decision variables, ΔR refers to the total number of charging station within R . To address spatial heterogeneity, for a particular location, r ($r \in R$), with an area infinitely close to 0, it is assumed that a value specified at the centroid of r reflects an average situation nearby, and the periphery is relatively homogeneous. The other two decision variables are the capacity of each charging station C [chargers/station], and the number of fast chargers per station ε ($\varepsilon \in [0, C]$) . The average charging demand per hour for a unit area at r is defined by Λ_r [veh/hr/km^2], and the average travel speed on the road at r is v_r [km/hr].

Charging demand modelling

We estimate the average charging demand per hour for a unit area based on EV-related parameter values, adopted from (An et al., 2025):

$$\Lambda_r = (N_r \Big/ \frac{\varphi}{\gamma \cdot l}) \cdot f \frac{1}{o_r \cdot r} \tag{1}$$

where N_r [veh] represents the number of EVs in r ; φ [kWh] denotes the average amount of electricity an EV charges in a single charging session; γ [kWh/km] indicates the average energy consumption per kilometer; l [km/day] indicated the average daily travel distance; f [%] refers to the frequency of using shared charging stations; o_r [hr/day] defines the operational charging hour per day. In this context, $\frac{\varphi}{\gamma \cdot l}$ represents the average charging frequency per EV, and $N_r \Big/ \frac{\varphi}{\gamma \cdot l}$ refers to the average daily charging demand across all charging stations. Multiplying by $f \frac{1}{o_r \cdot r}$ converts this to the average charging demand per hour for a unit area, with a dimension in [veh/hr/km^2].

Travel time and waiting time modelling

For travel time in each particular area, it is assumed that EVs would choose a shared charging station with the shortest travel time to recharge and the availability of each station would be known in advance through the broadcast equipped on EVs, so the average distance from each EV to the target shared charging station is determined by the density of charging station, and can be

approximately calculated as $\frac{\sqrt{2}}{3}\frac{1}{\sqrt{\Delta_r}}$ when stations are positioned on a square lattice at 45 degrees (Daganzo and Ouyang, 2019). In this case, the expected travel time for each EV, t_{travel} [hr/veh] is calculated as:

$$t_{travel} = \frac{\sqrt{2}}{3}\frac{1}{v_r\sqrt{\Delta_r}} \tag{2}$$

which can be implicated that the travel time is only influenced by one decision variable Δ_r.

For waiting time calculation, the M/G/k queueing model is adopted based on the classic M/M/k queueing model to overcome the overestimation of the queueing time (Lee and Longton, 1959), with an approximation to estimate the expected waiting time in a queue, $\overline{W}_q^{M/G/k_r}$ [hr/veh], which can be calculate as follow:

$$\overline{W}_q^{M/G/k_r} \approx \frac{\left(\theta_r^{arr}\right)^2 + \left(\theta_r^{ser}\right)^2}{2}\overline{W}_q^{M/M/k_r} \tag{3}$$

where $\overline{W}_q^{M/M/k_r}$ [hr/veh] represents the expected waiting time in a queue using the M/M/k queueing model with the charger number of k_r at r; θ_r^{arr} and θ_r^{ser} refer to the coefficient variation of arrival and charging time, and can be represented as $\sigma_r^{arr}/\frac{1}{\lambda_r}$ and $\sigma_r^{ser}/\frac{1}{\mu_r}$, respectively. Here σ_r^{arr} and σ_r^{ser} are the standard deviation of arrival and charging time distribution, respectively. For the coefficient variation of arrival time θ_r^{arr}, σ_r^{arr} is equivalent to $\frac{1}{\lambda_r}$, as EV arrivals at the station follow a Poisson process, inherently resulting in $\theta_r^{arr}=1$. However, consider the fact that there are minimal and maximal limitations for EV charging process, we assume that the charging time follows a truncated normal distribution, resulting in $\mu_r^{ser,f}\approx 0.417$, $\sigma_r^{ser,f}\approx 0.095$ for 50 kW fast charger based on reference parameter and equations from (Burkardt, 2014), which can be scaled to 3.6 kW slow charger as $\mu_r^{ser,s}\approx 5.787$, $\sigma_r^{ser,s}\approx 1.326$ based on $\mu_r^{ser,s}=\frac{P_f\cdot\mu_r^{ser,f}}{P_s}$ and $\sigma_r^{ser,s}=\frac{P_f\cdot\sigma_r^{ser,f}}{P_s}$, then, for the mixed queueing system considered here, we assumed a equivalent charging rate $\mu_r^{ser,equ}$, standard deviation $\sigma_r^{ser,equ}$, and coefficient variation of charging time $\theta_r^{ser,equ}$ calculated as:

$$\mu_r^{ser,equ} = \frac{\varepsilon_r\cdot\mu_r^{ser,f} + \left(C_r-\varepsilon_r\right)\cdot\mu_r^{ser,s}}{C_r} \tag{4}$$

$$\sigma_r^{ser,equ} = \sqrt{\left\{\left(C_r-\varepsilon_r\right)\left[\left(\sigma_r^{ser,s}\right)^2+\left(\mu_r^{ser,s}\right)^2\right]+\varepsilon_r\left[\left(\sigma_r^{ser,f}\right)^2+\left(\mu_r^{ser,f}\right)^2\right]\right\}\Big/C_r-\left(\mu_r^{ser,equ}\right)^2} \tag{5}$$

$$\theta_r^{ser,equ} = \frac{\sigma_r^{ser,equ}}{\mu_r^{ser,equ}} \tag{6}$$

In the context of a multiple-server system, the occupancy ρ_r, which refers to the utilization of the servers, is represented as:

$$\rho_r = \frac{\lambda_r}{\mu_r^{ser,equ} \cdot C_r} < \rho_{\max} \tag{7}$$

where λ_r can be calculated by $\lambda_r = \frac{\Lambda_r}{\Delta_r}$, with a unit of [veh/hr/station], $\rho_{\max}$ is the maximal allowable occupancy and is generally set to 1, but it can be adjusted to a lower value, such as 0.9 or 0.8, to enhance the system stability. In this research, $\rho_{\max} = 0.8$. Using this term, the average waiting time in a queue $\overline{W}_q^{M/M/k_r}$ can be obtained through the following equation:

$$\overline{W}_q^{M/M/k_r} = \frac{P_0}{\lambda_r} \cdot \frac{\rho_r \cdot \left(k_r/\rho_r\right)^{k_r}}{k_r! \cdot \left(1-\rho_r\right)^2} \approx \frac{P_0}{\lambda_r} \cdot \frac{\rho_r \cdot \left(k_r/\rho_r\right)^{k_r}}{\Gamma\left(k_r+1\right) \cdot \left(1-\rho_r\right)^2} \tag{8}$$

where P_0 represents the probability of no queue, and can be calculated as:

$$P_0 = \left[\sum_{n=1}^{k_r-1} \frac{\left(k_r \cdot \rho_r\right)^n}{n!} + \frac{\left(k_r \cdot \rho_r\right)^{k_r}}{k_r! \cdot \left(1-\rho_r\right)}\right]^{-1} \approx \left[\sum_{n=1}^{k_r-1} \frac{\left(k_r \cdot \rho_r\right)^n}{\Gamma\left(n+1\right)} + \frac{\left(k_r \cdot \rho_r\right)^{k_r}}{\Gamma\left(k_r+1\right) \cdot \left(1-\rho_r\right)}\right]^{-1} \tag{9}$$

We use gamma function instead of factorial to handle continuous variables and to enable an optimization framework, such as $n! \approx \Gamma\left(n+1\right)$. It can be seen that the waiting time per EV expressed here is influenced by all three decision variables, Δ_r, C_r, and ε_r.

Mixed-integer nonlinear optimization problem formulation

Using the metrics defined in the CA framework, we can form a MINLP which involves a nonlinear objective function subject to multiple constraints, with both continuous and discrete decision variable as follows:

$$\begin{aligned} T_{spent}(\Delta, C, \varepsilon) &= \iint_{r \in R} \frac{\Lambda_r}{\iint_{r \in R} \Lambda_r dr} t_{spent}(\Delta_r, C_r, \varepsilon_r)\, dr \\ &= \iint_{r \in R} \frac{\Lambda_r}{\iint_{r \in R} \Lambda_r dr} \left\{t_{travel}(\Delta_r) + t_{waiting}(\Delta_r, C_r, \varepsilon_r)\right\}\, dr \end{aligned} \tag{10}$$

The total cost includes the cost for the installation of charging stations and chargers, denoted by $C\left(\Delta, C, \varepsilon\right)[€]$, must not exceed the allocated budget, and this budget constraint is expressed as:

$$\begin{aligned} C\left(\Delta, C, \varepsilon\right) &= \iint_{r \in R} c\left(\Delta_r, C_r, \varepsilon_r\right)\, dr \\ &= \iint_{r \in R} \Delta_r \cdot [\zeta + c_r \cdot \varepsilon_r \cdot \xi_f + (c_r - \varepsilon_r) \cdot \xi_s]\, dr \le B \end{aligned} \tag{11}$$

where $c(\Delta_r, C_r, \varepsilon_r)$ indicate the cost at r with station density Δ_r, station capacity C_r, and number of fast charger per station ε_r; ζ, ξ_f, and ξ_s refer to the cost per charging station, fast charger, and slow charger, respectively; while B denotes the allocated budget for constructing the charging network over the service region.

The total power contains the power needed to satisfied all charging demand connected to all types of chargers, denoted by $P(\Delta, C, \varepsilon)$ [kW], should be limited to the maximal power supply for the charging system within the service area, and this constrain is expressed as:

$$\begin{aligned} P(\Delta, C, \varepsilon) &= \iint_{r \in R} p_r(\Delta_r, C_r, \varepsilon_r) dr = \\ &= \iint_{r \in R} \Delta_r [\varepsilon_r \cdot \varphi_f + (C_r - \varepsilon_r) \cdot \varphi_s] dr \le (1-\sigma) \cdot P_{\max} \end{aligned} \tag{12}$$

where $p_r(\Delta_r, C_r, \varepsilon_r)$ denote the power needed to meet all the charging demand within a unit area; φ_f and φ_s refer to the power of fast and slow charger, respectively. σ is the margin of the power system which is normally 5%; $P_{\max}$ is the maximum power supply capacity of the power system in the planning area.

As the number of decision variables grows rapidly in the CA framework with the size of the research area, it is not free of difficulty to solve such a problem. So, we decided to divide the research area into finite number of zones, each of which is a collection of periphery elements with relatively homogenous properties. Therefore, the following mathematical programming can be present:

$$\min T_{spent}(\Delta, C, \varepsilon) = \min \sum_{r \in R} \frac{\Lambda_r \cdot r}{\sum_{r \in R} \Lambda_r \cdot r} \left\{ t_{travel}(\Delta_r) + t_{waiting}(\Delta_r, C_r, \varepsilon_r) \right\} \tag{13}$$

subject to

$$C(\Delta, C, \varepsilon) = \sum_{r \in R} r \Delta_r \left\{ \zeta + \varepsilon_r \cdot \xi_f + (C_r - \varepsilon_r) \cdot \xi_n \right\} \le B \tag{14}$$

$$P(\Delta, C, \varepsilon) = \sum_{r \in R} r \Delta_r \left[\varepsilon_r \cdot \varphi_f + (C_r - \varepsilon_r) \cdot \varphi_s \right] \le (1-\sigma) \cdot P_{\max} \tag{15}$$

$$\Delta_r \ge 0, \forall r \in R \tag{16}$$

$$\Delta_r \le \Delta_{r,\max}, \forall r \in R \tag{17}$$

$$c_r \ge 1, \forall r \in R \tag{18}$$

$$c_r \le c_{r,\max}, \forall r \in R \tag{19}$$

Differential evaluation has been adopted to search for optimal solutions under the constraints of budget and power supply. A converged Pareto frontier can be formed to analyze the trade-off between infrastructure cost and the system performance, to be specific, T_{spent}.

3. THE BARCELONA CASE STUDY

We apply the proposed CA-based model to the City of Barcelona, within the whole AMB (Àrea Metropolitana de Barcelona) area, as shown in Figure 1. In following subsections, we design the

case study first, then discuss the results obtained from the model to provide insights for shared charging station planning.

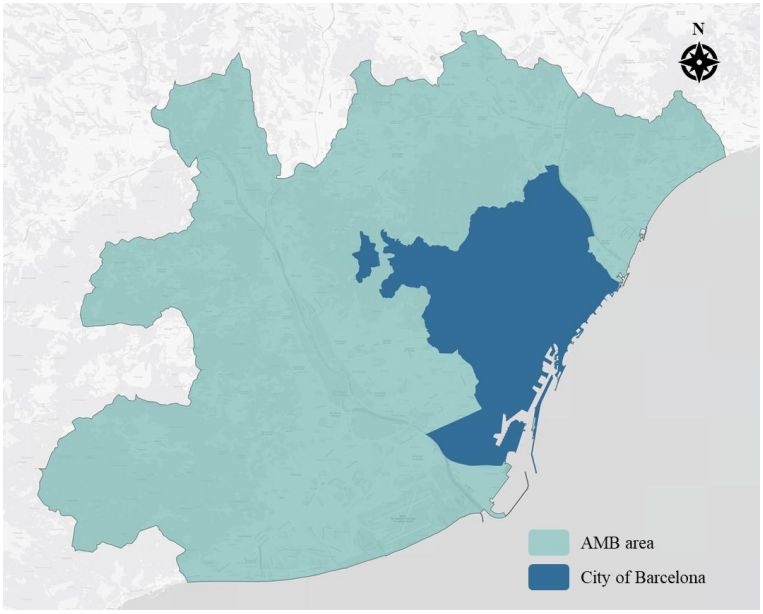


Figure 1: City of Barcelona

Case study design

Given the nature of CA framework, we decompose City of Barcelona into 10 boroughs. The data of shared charging stations, registered EVs within each borough, and area sizes can be obtained through Open Data BCN (https://opendata-ajuntament.barcelona.cat/es/) and the official website of AMB (https://www.amb.cat/s/web/area-metropolitana/municipis-metropolitans.html). As for the parameters related to EVs, 2023 Tesla Model 3 (https://www.tesla.com/model3/design#overview) has been chosen as the reference vehicle since it is the most popular EV in Barcelona. The parameter values used in this case study are summarized in Table 1, the current layout of shared charging stations and distribution of estimated charging demand are illustrated in Figure 2.

Table 1: Input parameters for the Barcelona case study

Parameter description	Notation	Units	Value	Source
Average travel speed in Barcelona	v	[km/hr]	30	https://www.barcelona.cat/mobilitat/en/barcelona-30-city
Average amount of electricity EV charges in a single session	φ	[kWh]	11.5	https://ev-database.org/
Average electricity consumption per kilometer	γ	[kWh/km]	0.135	
Average daily driving distance	l	[km/day]	50	https://endolla.barcelona/en/news/endolla-service/real-range-electric-car?utm_source=chatgpt.com
Average operational hour per day per charging station	o	[hr/day]	18	—
Power of fast charger	P_f	[kW]	50	https://opendata-ajuntament.barcelona.cat/es/
Power of slow charger	P_s	[kW]	3.6	
Cost of constructing a charging station	ζ	[€]	8000	https://www.beev.co/en/blog/borne-de-recharge/cout-installation-borne-entreprise/
Cost of installing a fast charger	ξ_f	[€]	36000	
Cost of installing a slow charger	ξ_s	[€]	5000	

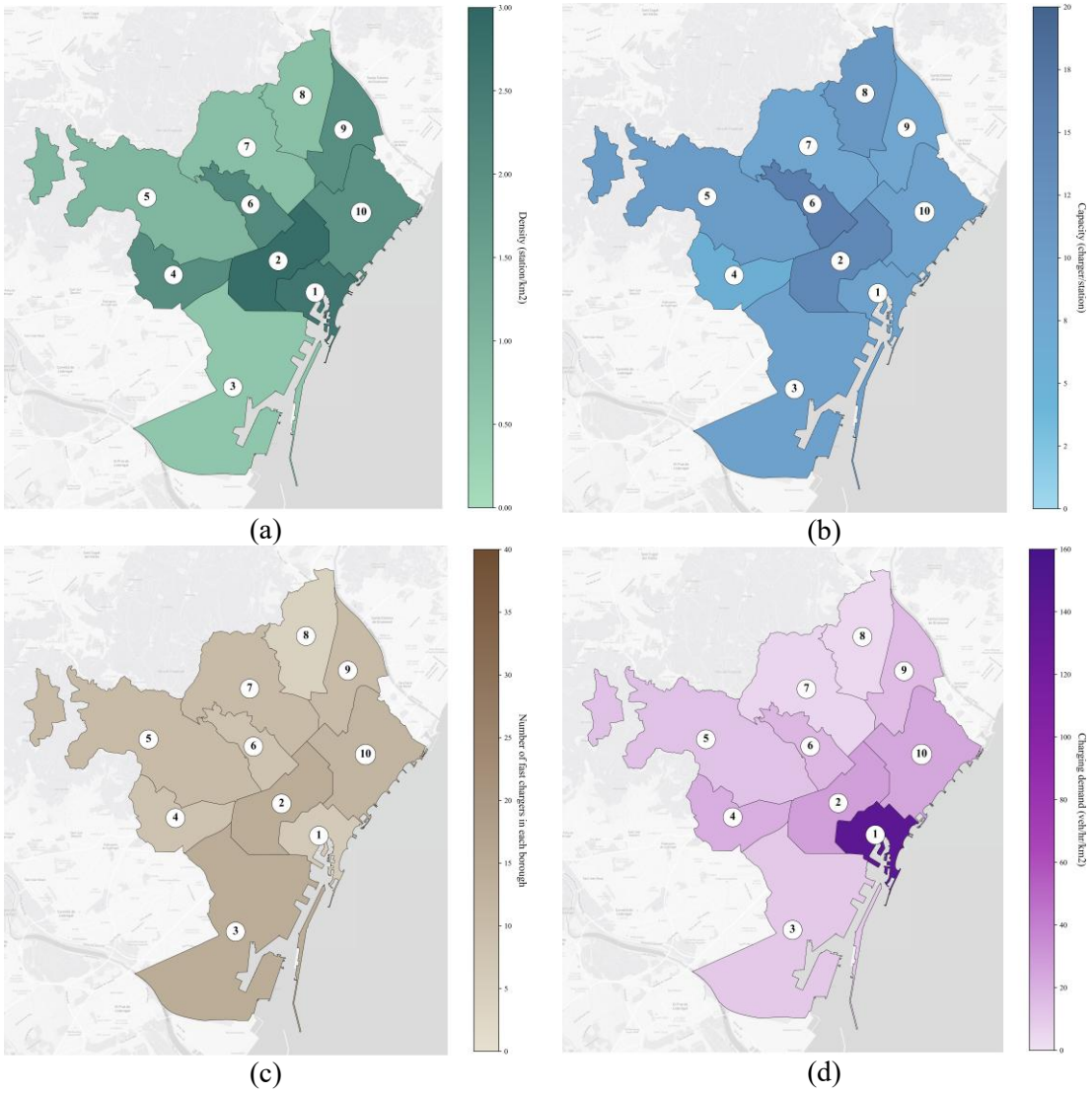


Figure 2: Current layout of shared charging stations in Barcelona (a) density, (b) capacity, (c) fast chargers, and (d) estimated charging demand per borough

Comparative analysis between optimal planning and status quo

Figure 3 illustrates the relationship between total time spent and investment budget using the proposed CA-based model. The light-blue line clearly shows that the total time spent decreases as the budget increases, however, this decrease slows down as the budget keeps rising and ends up constantly at 0.014 at a cost of 8.504 million euros investment. The gray point on the right side represents the total time spent and the investment budget of the status quo of Barcelona, a 0.023 hr/veh total time spent at the cost of 9.696 million euros station planning. The proposed planning model shows a 39.1% decrease in time spent and 12.3% decrease in investment budget, which can be promising for the development of EVs for both the users and charging service agencies. Besides, regarding the agency's side, insight can be drawn that the marginal return on time spent is getting smaller as the investment increases, which highlights the importance of increasing the utilization rate of existing facilities instead of spending more money to build new ones.

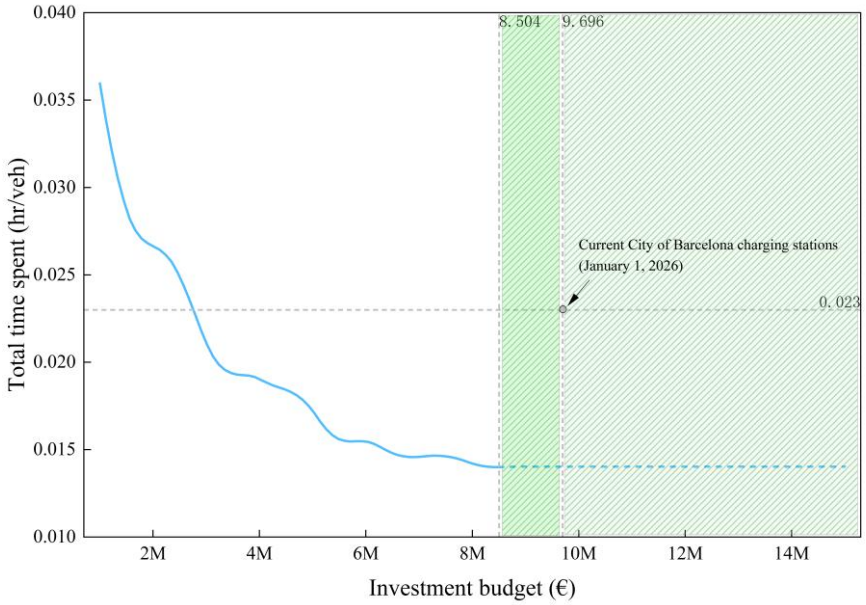


Figure 3: Trade-off between total time spent and investment budget and comparative analysis with status quo of City of Barcelona

Table 2: Comparation of charging infrastructure specifics

Borough ID	**Density-BAU** (station/km2)	**Density-Optimal** (station/km2)	**Capacity-BAU** (charger/station)	**Capacity-Optimal** (charger/station)	**Num. of fast charger-BAU**	**Num. of fast charger-Optimal**
Ciutat Vella ①	2.62	0.59	9	10	6	0
Eixample②	2.81	1.47	14	8	14	11
Sants-Montjuic③	0.57	0.86	9	9	14	20
Les Corts④	2.00	1.35	6	7	8	0
Sarria-Sant Gervasi⑤	0.95	0.84	10	9	10	17
Gracia⑥	2.13	1.46	16	9	8	31
Horta-Guinardo⑦	0.76	0.7	8	4	10	0
Nou Barris⑧	0.74	0.66	11	7	4	5
Sant Andreu ⑨	1.97	1.5	8	5	10	10
Sant Marti⑩	1.92	1.5	9	8	12	0

Table 2 shows the specifics of charging infrastructure before and after optimization. Results indicate that charging demand plays a decisive role in shaping the optimal charging infrastructure configuration. In ultra-high demand areas such as Ciutat Vella, the model reduces both station density and fast chargers, suggesting that congestion mitigation is achieved by stabilizing the service system rather than increasing capacity. In contrast, medium-demand regions such as

Gracia exhibit a substantial increase in fast chargers, indicating that enhancing service rates is the most effective way to reduce queueing delays. For low-demand areas, only marginal adjustments are observed due to the inherently low system utilization. These findings demonstrate that spatial heterogeneity affects not only the scale but also the structure of infrastructure investment, leading to differentiated strategies across regions.

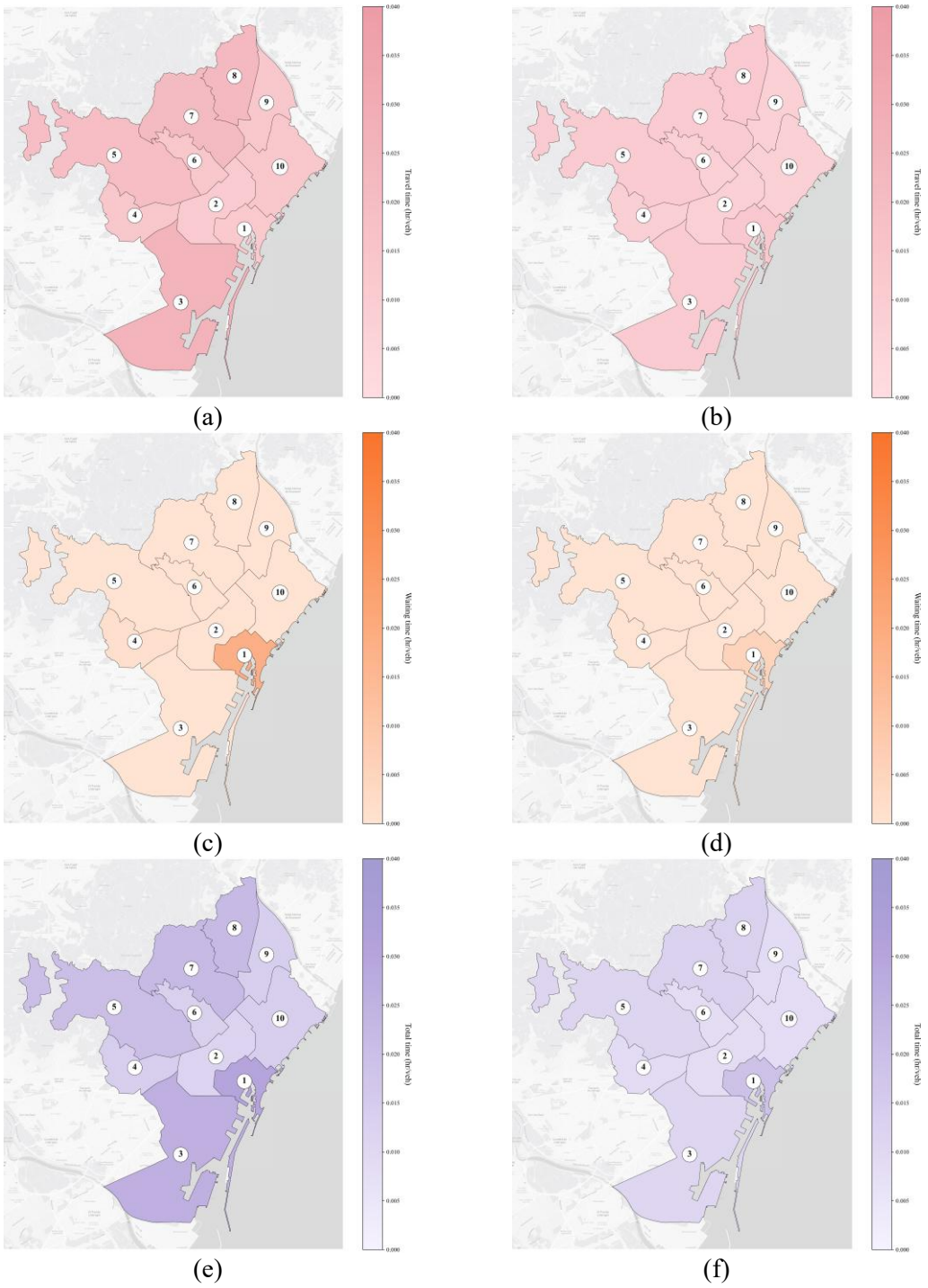


Figure 4: Distribution of time spent in each borough

Figure 4 illustrates the spatial distribution of travel time, waiting time and total time in each borough before (left) and after (right) optimization. It can be seen that travel time in almost every borough experienced a decrease compared with the status quo, the decrease ranges from 29% to 59%. Regarding waiting time, a 70% decrease can be found in Ciutat Vella where the highest charging demand existed, while other boroughs saw a rise in this metric, which can be seen as an outspreading of congestion after considering spatial heterogeneity in each borough. For total time spent in each borough, decrease can be found in every borough which further verifies the effectiveness of the proposed model in reducing time spent on charging related activities.

4. CONCLUSIONS

This study proposed a CA-based optimization model for the planning of shared charging station under spatial heterogeneity. By jointly determining station density, capacity, and the number of fast chargers per station, the model minimizes total time spent charging subject to budget and power supply constraints. For practical application, we implemented the proposed model to city of Barcelona. Results indicated that the proposed model can substantially reduce total time spent while requiring lower investment compared to the existing infrastructure layout. The results further highlight the critical role of spatial heterogeneity, showing that optimal configurations vary significantly across urban areas with different demand patterns. This research contributes to the literature by integrating heterogeneous charger types into shared charging station planning. From a practical perspective, the findings provide actionable insights for urban planners, emphasizing the importance of adaptive and location-specific deployment strategies rather than uniform expansion. Future work could extend the model by incorporating larger-scale networks, heterogeneous EV characteristics, and dynamic demand patterns to further enhance its applicability and robustness.

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