A Branch-and-Price Algorithm for Modular Autonomous Vehicle Routing Problem with Pickup and Delivery

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Short summary

The development of on-demand transportation and Modular Autonomous Vehicles (MAVs) presents opportunities for dynamic capacity adjustment. However, existing MAV routing models are primarily restricted to fixed transit corridors, limiting their full potential. To bridge this gap, this paper introduces a novel MAV routing problem with pickup and delivery in a fully flexible environment. We propose a model aimed at minimizing total travel costs and penalties for unserved requests, while explicitly incorporating the cost-saving platooning effects of docked MAVs. The problem is formulated as an arc-flow model and reformulated using Dantzig-Wolfe decomposition. To achieve optimal solutions, we develop a branch-and-price algorithm incorporating a labeling algorithm to solve the pricing subproblem. Numerical results show that our exact algorithm outperforms solver in terms of solution speed.

1 Introduction

The evolution of on-demand transportation has spurred two significant streams of research aimed at enhancing operational efficiency and service quality. One stream focuses on optimizing passenger-to-vehicle assignments through effective ride-sharing algorithms, while the other investigates the optimization of the vehicle itself through the use of MAVs.

Extensive research has focused on optimizing ride-sharing systems, which tackle the core challenges of real-time demand matching and vehicle routing (e.g., Bruglieri et al., 2024; de Palma et al., 2024; Dickerson et al., 2021; Fan et al., 2024; Masoud et al., 2017; Zhao et al., 2018). Another promising research stream investigates the use of MAVs to address the mismatch between fluctuating passenger demand and fixed vehicle capacity (Chen et al., 2022). The ability of MAVs to couple and decouple en-route offers dynamic capacity adjustment. Initial works have established the value of this flexibility, demonstrating an "option premium" for switching vehicle sizes (Guo et al., 2017) and developing foundational routing models for logistics applications (Zhou et al., 2025). Despite this potential, the application of MAVs has been conceptualized within similarly constrained transit environments. The majority of studies model MAVs operating along fixed transit corridors with a set of pre-determined stations for docking and undocking (Lin & Zhang, 2024; Tian et al., 2022, 2023). Even when more flexible designs are considered, such as customized bus networks (Gong et al., 2021) or hybrid services that deviate from a main route (Tang et al., 2024; Liu et al., 2021), the operational paradigm remains limited to a fixed network backbone. This focus on corridor-based models prevents MAVs from being deployed in a fully flexible, unconstrained setting where their full potential could be realized.

Ride-sharing often lacks capacity adaptability and is constrained by network structures, while the MAV research, despite offering capacity flexibility, has been largely confined to these very same structured networks. This research bridges this critical gap by developing a model for a MAV Routing Problem for Pickup and Delivery that integrates the strengths of both domains. This work is novel in its synthesis of MAVs with fully flexible routing in a many-to-many service paradigm and explicit formation operations in an unconstrained environment.

Problem statement

This paper considers routing a homogeneous fleet of MAVs for passenger pickup and delivery, aims to minimize the total travel cost and penalty cost charged for unserved passenger request. Fig. 1 illustrates the operation of two MAVs for five requests. Each request is associated with a pickup

node and a deliver node. If a request is not served, penalty cost will be charged for the MAV operation system. During the operation, MAVs may dock together to travel in platoon due to platooning effect. Fig. 2 illustrates the platooning effect in terms of arc travel cost with respect to platoon size. As the platoon size increases, the arc travel cost of each MAV in the platoon will decrease nonlinearly, where the platoon effect index ϑ controls the degree of reduction in travel cost.

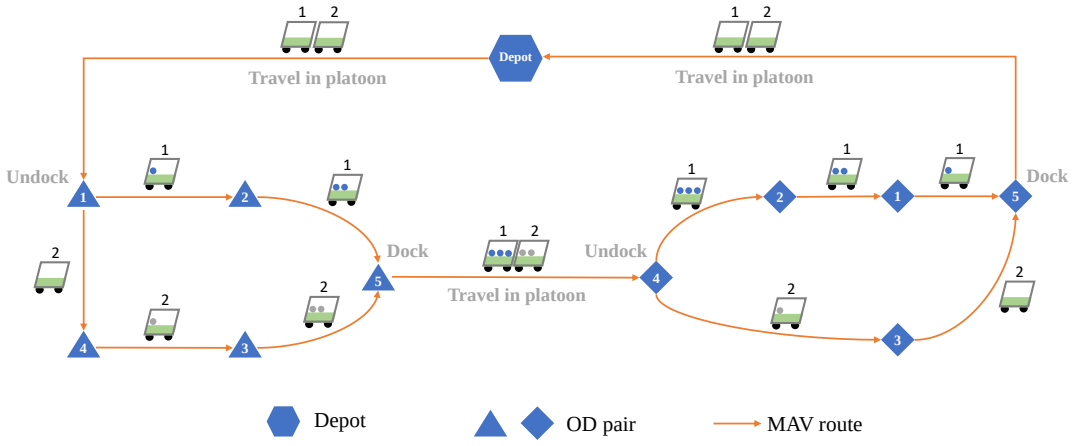


Figure 1: An example of routing two MAVs for five requests.

The Modular Autonomous Vehicle Routing Problem (MAVRP) can be defined on a directed graph $\mathbf{G} = (\mathbf{V}, \mathbf{A})$, where $\mathbf{V} = \{0, \ldots, 2n+1\}$ is the node set and $\mathbf{A}$ is the arc set. n denotes the number of requests to be served, node 0 and $2n+1$ denote the source and sink depot, respectively. Each arc $(i, j) \in \mathbf{A}$ is associated with an initial travel cost of $c_{i,j}$ and a travel time of $T_{i,j}$. We further define two subsets of node set, $\mathbf{P} = \{1, \ldots, n\}$ and $\mathbf{D} = \{n+1, \ldots, 2n\}$, as the set of pickup nodes and set of deliver nodes, respectively.
Each request i is associated with a pickup node i, a deliver node $n+i$, a load demand q_i, and a service time window $[\overline{e}_i, \overline{l}_i]$. Without loss of generality, we let $q_0 = q_{2n+1} = 0$ and $q_i = -q_{n+1}, i \in \mathbf{P}$. All MAVs are initially at the source depot 0 and return to the sink depot $2n+1$ upon completing tasks. We let $\mathbf{B}$ denote the set of available MAVs. Each MAV is associated with a load capacity of Q, and a maximum platoon size of L. During the delivery, MAVs are allowed to dock as platoons or undock as individual unit modules.

2 Methodology

In this section, we first formulate an arc-flow model for the MAVRP. Second, we reformulate the model into Master Problem and Pricing Subproblem via Danzig-Wolfe Decomposition. Finally, we present a branch-and-price algorithm for solving this problem optimally.

Arc-flow model formulation

We introduce five decision variables for the arc-flow formulation of MAVRP. Let α_i^m be a binary variable equal to 1 if the associated request of pickup node $i \in \mathbf{P}$ is served by MAV $m \in \mathbf{B}$, and 0 otherwise. Let $x_{i,j}^m$ be a binary variable equal to 1 if arc $(i, j) \in \mathbf{A}$ is traversed by MAV $m \in \mathbf{B}$, and 0 otherwise. Let $\tau_{i,j}^m$ be a nonnegative integer indicates the number of MAVs with MAV $m \in \mathbf{B}$ (including m itself) traversing on arc $(i, j) \in \mathbf{A}$. Let t_i^m be a continuous variable indicates the time at which MAV $m \in \mathbf{B}$ begins service at node $i \in \mathbf{V}$. Let h_i^m be a nonnegative integer measures the load of MAV $m \in \mathbf{B}$ when leaving node $i \in \mathbf{V}$. Therefore, the MAVRP can be formulated as an arc-flow model as in (1a)-(1z).

$$\min \sum_{l=1}^{L} \sum_{(i,j)\in\mathbf{A}} c_{i,j} \cdot l \cdot F(l, \vartheta) \cdot z_{i,j}^l + c^R \cdot \sum_{i\in\mathbf{P}} \left(1 - \sum_{m\in\mathbf{B}} \alpha_i^m\right) \tag{1a}$$

s.t.

$$\sum_{m\in\mathbf{B}} \alpha_i^m \leq 1 \quad \forall i \in \mathbf{P}, \tag{1b}$$

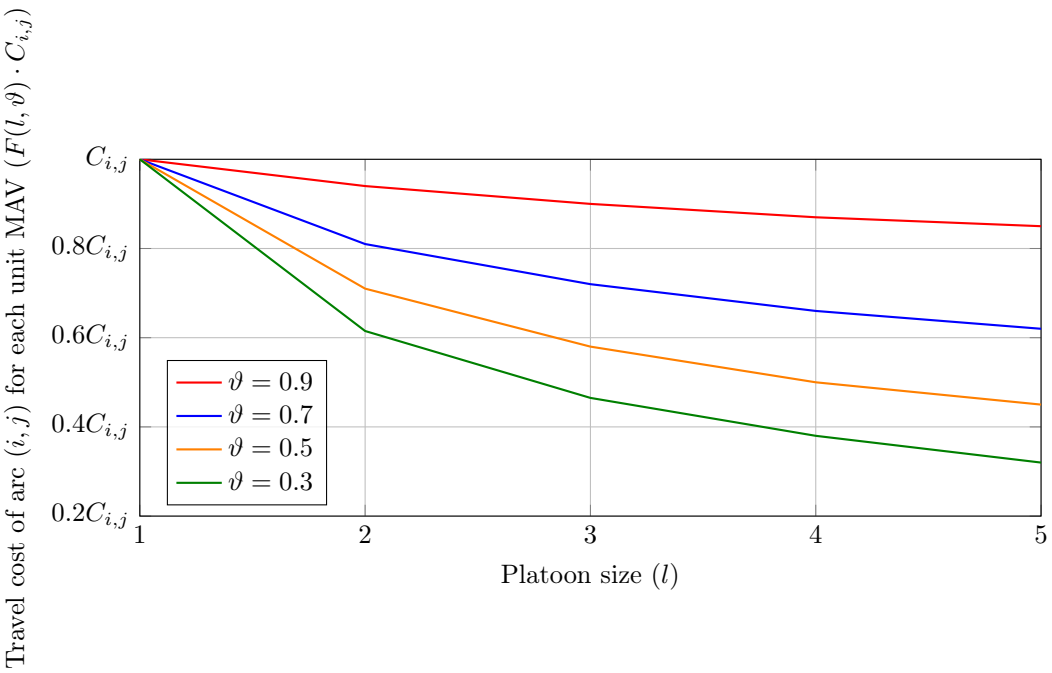


Figure 2: Arc travel cost for unit MAV with different platoon size under different energy indexes.

$$\sum_{j\in\mathbf{V}^+(i)} x_{i,j}^m \geq \alpha_i^m \quad \forall i \in \mathbf{P}, m \in \mathbf{B}, \tag{1c}$$

$$\sum_{j\in\mathbf{V}^+(n+i)} x_{n+i,j}^m \geq \alpha_i^m \quad \forall i \in \mathbf{P}, m \in \mathbf{B}, \tag{1d}$$

$$\sum_{j\in\mathbf{V}^+(0)} x_{0,j}^m = 1 \quad \forall m \in \mathbf{B}, \tag{1e}$$

$$\sum_{i\in\mathbf{V}^-(2n+1)} x_{i,2n+1}^m = 1 \quad \forall m \in \mathbf{B}, \tag{1f}$$

$$\sum_{i\in\mathbf{V}^-(j)} x_{i,j}^m - \sum_{i\in\mathbf{V}^+(j)} x_{j,i}^m = 0 \quad \forall j \in \mathbf{P}\cup\mathbf{D}, m \in \mathbf{B}, \tag{1g}$$

$$\sum_{m\in\mathbf{B}} \sum_{j\in\mathbf{V}^+(i)} x_{i,j}^m \geq \sum_{m\in\mathbf{B}} \alpha_i^m \quad \forall i \in \mathbf{P}, \tag{1h}$$

$$\sum_{m\in\mathbf{B}} \sum_{j\in\mathbf{V}^+(i)} x_{i,j}^m \leq L \cdot \sum_{m\in\mathbf{B}} \alpha_i^m \quad \forall i \in \mathbf{P}, \tag{1i}$$

$$\sum_{l=1}^{L} l \cdot z_{i,j}^l = \sum_{m\in\mathbf{B}} x_{i,j}^m \quad \forall (i,j) \in \mathbf{A}, \tag{1j}$$

$$\sum_{l=1}^{L} z_{i,j}^l \leq 1 \quad \forall (i,j) \in \mathbf{A}, \tag{1k}$$

$$t^m_{n+i} \geq t^m_i + T_{i,n+i} - M(1-\alpha^m_i) \quad \forall i \in \mathbf{P}, m \in \mathbf{B}, \tag{1l}$$

$$t^m_j \geq t^m_i + T_{i,j} - M(1-x^m_{i,j}) \quad \forall (i,j) \in \mathbf{A}, m \in \mathbf{B}, \tag{1m}$$

$$t^m_i \leq t^{m'}_i + M(2-x^m_{i,j}-x^{m'}_{i,j}) \quad \forall (i,j) \in \mathbf{A}, m \in \mathbf{B}, m' \in \mathbf{B}, \tag{1n}$$

$$t^m_i \geq t^{m'}_i - M(2-x^m_{i,j}-x^{m'}_{i,j}) \quad \forall (i,j) \in \mathbf{A}, m \in \mathbf{B}, m' \in \mathbf{B}, \tag{1o}$$

$$\bar{e}_i \leq t^m_i \leq \bar{l}_i \quad \forall i \in \mathbf{P} \cup \mathbf{D}, m \in \mathbf{B}, \tag{1p}$$

$$h^m_j \leq h^m_i + \alpha^m_j \cdot q_j + M(1-x^m_{i,j}) \quad \forall i \in \mathbf{V}, j \in \mathbf{P} \cap \mathbf{V}^+(i), m \in \mathbf{B}, \tag{1q}$$

$$h^m_j \geq h^m_i + \alpha^m_j \cdot q_j - M(1-x^m_{i,j}) \quad \forall i \in \mathbf{V}, j \in \mathbf{P} \cap \mathbf{V}^+(i), m \in \mathbf{B}, \tag{1r}$$

$$h^m_j \leq h^m_i + \alpha^m_{j-n} \cdot q_j + M(1-x^m_{i,j}) \quad \forall i \in \mathbf{V}, j \in \mathbf{D} \cap \mathbf{V}^+(i), m \in \mathbf{B}, \tag{1s}$$

$$h^m_j \geq h^m_i + \alpha^m_{j-n} \cdot q_j - M(1-x^m_{i,j}) \quad \forall i \in \mathbf{V}, j \in \mathbf{D} \cap \mathbf{V}^+(i), m \in \mathbf{B}, \tag{1t}$$

$$0 \leq h^m_i \leq Q \quad \forall i \in \mathbf{V}, m \in \mathbf{B}, \tag{1u}$$

$$\alpha^m_i \in \{0,1\} \quad \forall i \in \mathbf{P}, m \in \mathbf{B}, \tag{1v}$$

$$x^m_{i,j} \in \{0,1\} \quad \forall (i,j) \in \mathbf{A}, m \in \mathbf{B}, \tag{1w}$$

$$z^l_{i,j} \in \{0,1\} \quad \forall (i,j) \in \mathbf{A}, l \in \{1,\ldots,L\}, \tag{1x}$$

$$t^m_i \in \mathbb{R}_{\geq 0} \quad \forall i \in \mathbf{V}, m \in \mathbf{B}, \tag{1y}$$

$$h^m_i \in \mathbb{Z}_{\geq 0} \quad \forall i \in \mathbf{V}, m \in \mathbf{B}. \tag{1z}$$

The objective function (1a) minimizes the total cost of traversed route plus the penalty cost incurred by requests not being served. The first term represents the cost of traversed routes for each unit MAV, where an increase in the number of MAVs in a platoon results in a lower travel cost for each MAV through an arc, and $F(l,\vartheta)$ represents the coefficient of platooning effect when platoon size is l and platoon effect index is ϑ, as shown in Fig. 2. The second term represents the penalty cost for requests that have not been picked up. Constraints (1b) assign each request to at most one MAV, while (1c) and (1d) ensure the assigned MAV services both the corresponding pickup and delivery nodes. Constraints (1e) and (1f) mandate that MAVs originate from the source depot and return to the sink depot, and (1g) enforces flow conservation. For serviced requests, (1h) and (1i) limit the number of visiting MAVs to between one and L. Constraints (1j) and (1k) calculate and restrict vehicle flow on any given arc. Constraints (1l) guarantee delivery occurs after pickup, and (1m) maintain chronological service start times across consecutive nodes. When multiple MAVs platoon on the same arc, (1n) and (1o) synchronize their service commencement. Constraints (1p) enforce specified time windows. Finally, (1q)-(1t) track MAV loads, (1u) enforces capacity limits, and (1v)-(1z) specify variable domains.

Dantzig-Wolfe decomposition

Dantzig-Wolfe decomposition (Dantzig & Wolfe, 1960) solves large-scale linear programs by reformulating them into a master problem and independent subproblems. The master problem expresses the solution as a convex combination of the subproblems' extreme points or rays. Because the number of variables is often vast, it is solved iteratively via column generation. A restricted master problem (RMP) is solved using a limited subset of variables. Its dual values generate a new objective for the pricing subproblem, which identifies new variables (columns) with negative reduced costs. These columns are added to the RMP until no negative reduced cost variables exist, proving optimality. Embedding this column generation process within a branch-and-bound framework for integer programs yields the branch-and-price method (Desaulniers et al., 1998). We apply this decomposition to the MAVRP model (1b)-(1z), detailing the resulting master problem and pricing subproblem formulations.

To solve the MAVRP with branch-and-price, we use an extended set-partitioning formulation for the master problem with following notations. We denote Ω as the set of feasible routes, $\tilde{\mathbf{A}}$ as the set of arcs included in the solution, $\mathbf{P}$ as the set of pickup nodes, and N as the total number of available MAVs. To characterize vehicle route, we introduce two binary parameter: $X_{i,j}^{p}$ equal to 1 if route $p \in \Omega$ traverses arc $(i,j) \in \tilde{\mathbf{A}}$, and 0 otherwise; a_{ip} equal to 1 if associated request of pickup node $i \in \mathbf{P}$ is served by route $p \in \Omega$, and 0 otherwise. We introduce three binary decision variables: y_p equal to 1 if route $p \in \Omega$ is included in the solution, and 0 otherwise; $z_{i,j}^{l}$ equal to 1 if the vehicle flow of arc $(i,j) \in \tilde{\mathbf{A}}$ is $l \in \{1,\dots,L\}$, and 0 otherwise; θ_i equal to 1 if associated request of pickup node $i \in \mathbf{P}$ is served, and 0 otherwise. Then, the master problem for MAVRP is presented in (2a)-(2j).

[Master Problem]

$$\min \sum_{p\in\Omega} c_p^B \cdot y_p + \sum_{(i,j)\in\tilde{\mathbf{A}}} c_{i,j}^P \cdot l \cdot F(l) \cdot z_{i,j}^l + \sum_{i\in\mathbf{P}} c^R \cdot \theta_i \tag{2a}$$

s.t.

$$\sum_{p\in\Omega} y_p \le N \tag{2b}$$

$$\sum_{p\in\Omega} a_{ip} \cdot y_p + \theta_i = 1 \quad \forall i \in \mathbf{P}, \tag{2c}$$

$$\sum_{p\in\Omega} \sum_{j\in\mathbf{V}^+(i)} X_{i,j}^p \cdot y_p \ge 1 - \theta_i \quad \forall i \in \mathbf{P}, \tag{2d}$$

$$\sum_{p\in\Omega} \sum_{j\in\mathbf{V}^+(i)} X_{i,j}^p \cdot y_p \le L \cdot (1 - \theta_i) \quad \forall i \in \mathbf{P}, \tag{2e}$$

$$\sum_{p\in\Omega} X_{i,j}^p \cdot y_p = \sum_{l=1}^{L} l \cdot z_{i,j}^l \quad \forall (i,j) \in \tilde{\mathbf{A}}, \tag{2f}$$

$$\sum_{l=1}^{L} z_{i,j}^l \le 1 \quad \forall (i,j) \in \tilde{\mathbf{A}}, \tag{2g}$$

$$y_p \in \{0,1\} \quad \forall p \in \Omega, \tag{2h}$$

$$\theta_i \in \{0,1\} \quad \forall i \in \mathbf{P}, \tag{2i}$$

$$z_{i,j}^l \in \{0,1\} \quad \forall (i,j) \in \tilde{\mathbf{A}}, l \in \{1,\dots,L\}. \tag{2j}$$

The objective function (2a) aims to minimize the sum of based travel cost of routes, additive platoon cost and penalty of requests not served. The based travel cost of route p is defined as $c_p^B = \sum_{(i,j)\in p} c'_{i,j} \cdot X_{i,j}^p$, where $c'_{i,j}$ is the based travel cost of arc (i,j). The additive platoon cost of an arc (i,j) is defined as $c_{i,j}^P \cdot l \cdot F(l)$, where $c_{i,j}^P$ is the marginal platoon cost of arc (i,j), $F(l)$ is the cost reduction rate function with respect to vehicle flow l. Constraints (2b) limit the number of dispatched MAVs, while (2c) ensure all requests are either served or left open. Platoon sizes during pickups are regulated by (2d) and (2e). Finally, (2f) and (2g) calculate MAV arc flows, and (2h)-(2j) establish the variable domains.
To verify whether a solution of the master problem feasible in terms of traversed arc flow, we define a feasibility problem. Formally, given an integer solution of the master problem, let Ω' be the set of paths included in the solution, N_p and A_p be the set of nodes and arcs within the path $p \in \Omega'$, respectively. We introduce a nonnegative continuous variable $\overline{t}_i^p$ indicates the feasible service start time at node $i \in N_p$ of route $p \in \Omega'$. Given this notation, we define the following feasibility problem as in (3a)-(3f).

[Feasibility Problem]

$$\min 0 \tag{3a}$$

s.t.

$$\overline{t}_i^{p_1} \geq \overline{t}_i^{p_2} - M\left(1 - X_{i,j}^{p_1} \cdot X_{i,j}^{p_2}\right) \quad \forall (i,j) \in \tilde{A}, p_1 \in \Omega', p_2 \in \Omega', \tag{3b}$$

$$\overline{t}_i^{p_1} \leq \overline{t}_i^{p_2} + M\left(1 - X_{i,j}^{p_1} \cdot X_{i,j}^{p_2}\right) \quad \forall (i,j) \in \tilde{A}, p_1 \in \Omega', p_2 \in \Omega', \tag{3c}$$

$$\overline{e}_i \leq \overline{t}_i^p \leq \overline{l}_i \quad \forall i \in N_p, p \in \Omega', \tag{3d}$$

$$\overline{t}_j^p \geq \overline{t}_i^p + T_{i,j} \quad \forall (i,j) \in A_p, p \in \Omega', \tag{3e}$$

$$\overline{t}_i^p \in \mathbb{R}_{\geq 0} \quad \forall i \in N_p, p \in \Omega'. \tag{3f}$$

Master problem solutions may include arcs traversed by multiple vehicles, which must platoon to reduce travel costs. According to our problem settings that platooning requires simultaneous arc traversal, hence, an untargeted feasibility problem (3a) is solved to verify time feasibility. Constraints (3b) and (3c) synchronize service start times for shared arcs. Furthermore, (3d) ensures these adjusted times respect specified time windows, (3e) maintains the chronological relationship between nodes, and (3f) establishes the variable domains.
To find promising columns to add to the restricted version of model (2), we solve one subproblem for each MAV $m \in \mathbf{B}$. That is, for each MAV m, we try to find a route p that has a negative reduced cost given by $\overline{c}'_p = c'_{i,j} - \varpi - \sum_{i\in\mathbf{P}} a_{ip} \cdot \pi_i - \delta_i^+ - \delta_i^- - \mu_{i,j}$, where ϖ, π_i, δ_i^+, δ_i^- and $\mu_{i,j}$ are the dual variables associated with constraints (2b), (2c), (2d), (2e) and (2f), respectively. The variables used in the pricing subproblem are similar to the variables in model (1) but ignoring the superscript m. The pricing subproblem is defined in (4a)-(4r).

[Pricing Subproblem]

$$\min \sum_{(i,j)\in\mathbf{A}} \tilde{c}_{i,j} \cdot x_{i,j} - \varpi \tag{4a}$$

s.t.

$$\sum_{j\in\mathbf{V}^+(i)} x_{i,j} \geq \alpha_i \quad \forall i \in \mathbf{P}, \tag{4b}$$

$$\sum_{j\in\mathbf{V}^+(n+i)} x_{n+i,j} \geq \alpha_i \quad \forall i \in \mathbf{P}, \tag{4c}$$

$$\sum_{j \in \mathbf{V}^+(0)} x_{0,j} = 1 \tag{4d}$$

$$\sum_{i \in \mathbf{V}^-(2n+1)} x_{i,2n+1} = 1 \tag{4e}$$

$$\sum_{i \in \mathbf{V}^-(j)} x_{i,j} - \sum_{i \in \mathbf{V}^+(j)} x_{j,i} = 0 \quad \forall j \in \mathbf{P} \cup \mathbf{D}, \tag{4f}$$

$$t_j \geq t_i + T_{i,j} - M\,(1 - x_{i,j}) \quad \forall (i,j) \in \mathbf{A}, \tag{4g}$$

$$t_{n+i} \geq t_i + T_{i,n+i} - M\,(1 - a_i) \quad \forall i \in \mathbf{P}, \tag{4h}$$

$$\overline{e}_i \leq t_i \leq \bar{l}_i \quad \forall i \in \mathbf{V}, \tag{4i}$$

$$h_j \leq h_i + \alpha_j \cdot q_j + M\,(1 - x_{i,j}) \quad \forall i \in \mathbf{V}, j \in \mathbf{P} \cap \mathbf{V}^+(i), \tag{4j}$$

$$h_j \geq h_i + \alpha_j \cdot q_j - M\,(1 - x_{i,j}) \quad \forall i \in \mathbf{V}, j \in \mathbf{P} \cap \mathbf{V}^+(i), \tag{4k}$$

$$h_j \leq h_i + \alpha_{j-n} \cdot q_j + M(1 - x_{i,j}) \quad \forall i \in \mathbf{V}, j \in \mathbf{D} \cap \mathbf{V}^+(i), \tag{4l}$$

$$h_j \geq h_i + \alpha_{j-n} \cdot q_j - M(1 - x_{i,j}) \quad \forall i \in \mathbf{V}, j \in \mathbf{D} \cap \mathbf{V}^+(i), \tag{4m}$$

$$0 \leq h_i \leq Q \quad \forall i \in \mathbf{V}, \tag{4n}$$

$$\alpha_i \in \{0,1\} \quad \forall i \in \mathbf{P}, \tag{4o}$$

$$x_{i,j} \in \{0,1\} \quad \forall (i,j) \in \mathbf{A}, \tag{4p}$$

$$h_i \in \mathbb{Z}_{\geq 0} \quad \forall i \in \mathbf{V}, \tag{4q}$$

$$t_i \in \mathbb{R}_{\geq 0} \quad \forall i \in \mathbf{V}, \tag{4r}$$

The objective function (4a) is aimed to minimize the reduced cost of a route, where $\tilde{c}_{i,j} = c'_{i,j} - \delta_i^+ - \delta_i^- - \pi_i - \mu_{i,j}$ if $i \in \mathbf{P}$ and $\alpha_i = 1$, and $\tilde{c}_{i,j} = c'_{i,j} - \mu_{i,j}$ otherwise. Constraints (4b)-(4f) are equivalent to (1c)-(1g). Constraints (4g), (4h) and (4i) are equivalent to (1l), (1m) and (1p), respectively. Constraints (4j)-(4n) are equivalent to (1q)-(1u).

Branch-and-Price algorithm

We solve the integer master problem (2) using a branch-and-price algorithm, which relies on column generation for lower bounds and a hierarchical branching strategy for fractional variables. Because the variable space is excessively large, we iteratively solve a restricted master problem (RMP) with a limited column subset. The RMP's dual values feed into a pricing subproblem, solved via a labeling algorithm, to identify and add negative reduced-cost columns until convergence.The labeling algorithm builds feasible routes on graph $\mathbf{G} = (\mathbf{V}, \mathbf{A})$ from source depot 0 to any node $i \in \mathbf{V}$ for MAV $m \in \mathbf{B}$. Each partial route is tracked by a label $L_i = \{P_i, C_i, \tilde{T}_i, UR_i\}$. Here, P_i represents the picked-up requests, C_i is the reduced cost, $\tilde{T}_i$ is the earliest service start time, and UR_i contains unreachable or already served nodes. A label L_i extends along valid physical arcs (i,j) (where $j \notin UR_i$) to form $L_j = \{P_j, C_j, \tilde{T}_j, UR_j\}$ via resource extension functions as follows.

$$P_j = \begin{cases} P_i \cup \{j\} & \text{if } \ j \in \mathbf{P} \\ P_i \backslash \{r\} & \text{if } \ j \in \mathbf{D} \\ P_i & \text{otherwise} \end{cases} \tag{5a}$$

$$C_j = C_i + \tilde{c}_{i,j} \tag{5b}$$

$$\tilde{T}_j = \max\left(\tilde{T}_i + T_{i,j}, \overline{e}_j\right) \tag{5c}$$

$$UR_j = UR_i \cup \{j\} \tag{5d}$$

To keep the search tractable, suboptimal paths are pruned using dominance conditions, which is given two labels $L_i^1 = \left\{P_i^1, C_i^1, \tilde{T}_i^1, UR_i^1\right\}$ and $L_i^2 = \left\{P_i^2, C_i^2, \tilde{T}_i^2, UR_i^2\right\}$ which are extended to the same node along two partial routes, L_i^1 dominates L_i^2 if $P_i^1 \subseteq P_i^2$, $C_i^1 \leq C_i^2$, $\tilde{T}_i^1 \leq \tilde{T}_i^2$, and $UR_i^1 \subseteq UR_i^2$.
When branching is required at a node of the branch-and-bound search tree, four types of branching decisions can be imposed, namely, on the total number of vehicles used, on the number of vehicles visiting each customer, on the total flow on each arc, and on the possibility of using or not using two arcs consecutively.

3 Results and discussion

Although the branch-and-price algorithm is not yet fully developed, we still conduct some preliminary computational experiments with Solomon datasets to reveal the performance of algorithm compared with solver. Table 1 presents the results obtained by branch-and-price and Gurobi. The performance of the algorithm is still far from what I expected, but the algorithm computation speed is much faster than Gurobi. In some cases, Gurobi cannot even obtain a valid upper bound within an hour, while our algorithm can obtain the optimal solution in less than 13 minutes.

Table 1: Performance of branch-and-price algorithm compared with solver

Instance	Branch-and-Price				Gurobi			
	LB	UB	Gap(%)	Time (s)	LB	UB	Gap(%)	Time (s)
RC-3-0.9	49.1	49.1	0.00	92	49.1	49.1	0.00	226
RC7-3-0.9	78.2	78.2	0.00	202	78.2	78.2	0.00	1,517
RC10-3-0.9	165.8	165.8	0.00	628	107.4	233.2	53.96	3,600
RC15-3-0.9	223.2	223.2	0.00	1,736	—	—	—	3,600
RC20-3-0.9	358.7	358.7	0.00	2,193	—	—	—	3,600
RC5-5-0.8	40.8	40.8	0.00	126	40.8	40.8	0.00	385
RC7-5-0.8	63.6	63.6	0.00	288	57.9	82.6	29.92	2,629
RC10-5-0.8	123.9	123.9	0.00	761	92.1	—	—	3,600
RC15-5-0.8	198.2	198.2	0.00	2933	—	—	—	3,600
RC20-5-0.8	315.8	315.8	0.00	3204	—	—	—	3,600

This work is still ongoing to improve the efficiency of the algorithm. At the next step, we will work on improving the algorithm with tailored branching strategies to efficiently handle the large number of fractional variables generated by the linear relaxation of RMP, and effective valid inequalities add to the RMP to tighten the linear relaxation. More computational experiments will be conducted to demonstrate the efficiency of the algorithm. Furthermore, numerical analysis will be employed to reveal the benefits of introducing MAVs for on-demand mobility service.

4 Conclusions

This study successfully develops a comprehensive routing model for a homogeneous fleet of Modular Autonomous Vehicles (MAVs) providing many-to-many passenger pickup and delivery services. By transitioning MAV operations from constrained, fixed-corridor networks to a fully flexible environment, our approach maximizes the vehicles' dynamic capacity adjustments and platooning capabilities. We formulated the MAVRP as an arc-flow model and utilized Dantzig-Wolfe decomposition to separate it into a master problem and a pricing subproblem. To efficiently solve the model, we implemented a branch-and-price algorithm. Preliminary computational experiments

demonstrate that our proposed algorithm significantly outperforms the Gurobi solver in terms of computation speed. Notably, our algorithm obtained optimal solutions in under 13 minutes for instances where Gurobi failed to establish a valid upper bound within an hour.

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