Designing Integrated Fare Systems: A Pareto-Based Evaluation of Structural Complexity and Price-Distance Fairness

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Short summary

This paper presents a decision-support framework for designing integrated fare systems across fixed-route and on-demand public transport. Using one year of operational and ridership data from Ingolstadt (Germany), we generate a set of ring- and cell-based zoning schemes and calibrate fares to preserve current single-ticket revenue.
For the evaluation, we focus on the trade-off between an easy-to-understand tariff system, captured by structural complexity, and price-distance fairness (Gini index). Fare-system performance is first assessed separately for fixed-route and on-demand services. In a second step, Pareto-efficient joint fare systems are identified and ranked by their weighted Euclidean distance to an ideal point representing minimum complexity and maximum fairness. Findings show that moderate zoning granularity can improve both simplicity and price-distance fairness. Overall, the framework provides transport authorities a decision-support tool for selecting user-oriented integrated fare systems.

Keywords: Fare System Pricing, Integrated Public Transport Systems, Multi-Modal Transport, Demand-Responsive Transport, Pareto Analysis

1 Introduction

Public transport systems increasingly integrate on-demand services as a complementary layer to fixed-route networks to improve coverage and cost efficiency in areas where traditional fixed-route services face insufficient demand. Recent research shows that such combinations can prevent user losses and attract additional demand by offering more flexible and higher-quality travel options (Fielbaum, Jara-Díaz, & Alonso-Mora, 2024).

An important question regarding integration is how to achieve a joint fare system that is easy to understand while maintaining price-distance fairness. Simplified and consistent fare systems reduce cognitive effort, lower perceived barriers to use public transport, and support system-wide attractiveness (Dydkowski & Urbanek, 2023). However, many existing fare systems, especially wide-spread zone-based systems with numerous boundaries between zones, introduce substantial complexity when passengers evaluate travel alternatives and reduce transparency of travel costs. This may negatively influence passengers' willingness to use public transport (Anciaes et al., 2019). These effects are in particular relevant for on-demand transport, which is typically valued for flexibility and ease of use (Buics et al., 2025). Simply embedding on-demand services into complex legacy tariffs of public transport therefore risks behavioral distortions.

From an operational standpoint joint consideration of on-demand and traditional public transport tariffs is feasible. In many regions, both services are planned or financed by the same public authority, but still follow separate tariff logics. This split can create inconsistent price signals, weaken the willingness to take multi-modal trips, and produce perceived unfairness between comparable trips. It may also limit the role of on-demand services as feeders to high-capacity fixed-route corridors,

reducing overall network efficiency. A coordinated fare architecture therefore supports not only simplicity for users, but also increases the attractiveness for multi-modal trips.

The design of public transport fare systems has been studied extensively, covering flat, distance-, and zone-based structures as well as multi-objective and behavioral optimization approaches (Hamacher & Schöbel, 2004; Müller et al., 2022; Otto & Boysen, 2017; Schiewe et al., 2024; Schöbel & Urban, 2025). However, a substantial portion of this literature addresses single-mode networks and fare optimization in conventional fixed-route services.

In contrast, research on integrating on-demand services into public transport mainly focuses on operational aspects such as service design, fleet sizing, routing, and network coordination (Calabrò et al., 2023; Fielbaum, Jara-Díaz, & Alonso-Mora, 2024; Fielbaum, Tirachini, & Alonso-Mora, 2024; Wu et al., 2025). Studies that incorporate fares typically optimize a specific integrated network configuration (Guo et al., 2025; Luo et al., 2021) or apply demand management (Anzenhofer et al., 2025) and dynamic pricing (Chen et al., 2020), but they rarely compare alternative tariff architectures across a broad design space or evaluate both service types jointly under the same assessment framework.

Against this background, this paper asks how integrated zoning-based fare systems can be designed to reduce structural complexity while preserving price-distance fairness across fixed-route and on-demand services. Using operational, spatial, and ridership data from the Ingolstadt region, we generate ring- and cell-based zoning schemes with different granularities and evaluate them in a unified framework. The analysis focuses on single-trip tickets; flat-rate products such as monthly subscriptions or the Deutschlandticket, Germany's nationwide public transport subscription ticket, are excluded, as their flat-rate logic leaves them unaffected by zone crossings.

The contribution of our work is threefold. First, we develop a data-driven procedure to generate and benchmark a broad design space of ring- and cell-based zoning systems (including a flat-fare benchmark). Second, we introduce an integrated, revenue-neutral assessment setup that evaluates each pricing scheme consistently across fixed-route and on-demand services using structural complexity (average zone crossings per trip) and price-distance fairness (Gini coefficient). Third, we provide a configurable decision-support framework in which weighting parameters explicitly represent stakeholder priorities between fairness and simplicity, and candidate schemes are selected using Pareto efficiency and distance-to-ideal rankings.

2 Methodology

In this section, we describe a stepwise decision-support procedure for integrated fare design. First, we generate a broad set of alternative ring- and cell-based zoning systems (plus a flat-fare benchmark and the current Ingolstadt zoning as reference). For each of these zoning scenarios, we derive fare levels for fixed-route and on-demand services that achieve the current revenue level. Second, we evaluate every scenario using structural complexity (average zone crossings) and price-distance fairness (Gini coefficient). Finally, we identify robust candidate systems by comparing them to the status quo, constructing weighted joint performance scores, and selecting Pareto-efficient solutions with minimum distance to an ideal benchmark.

Construction of Zone-based Fare Systems

In our modeling framework, we examine zone-based fare systems in which the service area is partitioned into a finite set of geographic zones. The implementation is stop-based, each stop is assigned to exactly one fare zone. A flat fare corresponds to assigning all stops to one zone.

A common and widely used principle to derive tariff steps in a zone-based fare system is using the *counting zones principle*, in which the fare depends solely on the number of zones a passenger crosses during the trip, independent of the specific OD pair (Hamacher & Schöbel, 2004).

We implement two zoning approaches: *cell-based zoning* and *ring-based zoning*, both constructed using data-driven methods that incorporate movements from both fixed-route services and on-demand operations to reflect actual travel patterns.

In *cell-based zoning*, stops are grouped into K contiguous cells. The zoning problem minimizes a weighted sum of geographical and flow-based separation costs. Let Z denote a feasible zone assignment, and let $z_i(Z)$ be the zone of stop i under Z. We define

$$C_{\text{geo}}(Z) = \sum_{i,j} d_{ij}\, h_{ij}(Z), \qquad C_{\text{flow}}(Z) = \sum_{i,j} F_{ij}\, m_{ij}(Z), \tag{1}$$

where d_{ij} is the geodetic distance between stops i and j, F_{ij} the passenger flow between them. The indicator functions $h_{ij}(Z)$ and $m_{ij}(Z)$ distinguish two types of undesirable separations: $h_{ij}(Z) = 1$ identifies geographically close stop pairs assigned to different zones, contributing to spatial fragmentation, while $m_{ij}(Z) = 1$ marks stop pairs with substantial passenger interactions that are split across zones, disrupting important OD flows. The combined zoning problem is

$$Z^* = \arg\min_{Z} \Big[\lambda\, C_{\text{geo}}(Z) + (1-\lambda)\, C_{\text{flow}}(Z)\Big], \tag{2}$$

where $\lambda \in [0,1]$ controls the trade-off between spatial compactness and preserving high-volume OD flows within zones. This formulation yields zones that jointly reflect geographical proximity and observed mobility patterns.

In *ring-based zoning*, stops are assigned to zones according to their geodetic distance from the networks' mobility hub. The hub is chosen as the stop with the highest aggregated passenger flow. Let R denote a vector of ring radii. Each stop i is assigned to a zone based on the distance interval into which its geodetic distance d_i from the hub falls, i.e., a stop belongs to zone k if

$$R_{k-1} < d_i \leq R_k \tag{3}$$

The radii R are optimized such that the total flow crossing ring boundaries is minimized:

$$R^* = \arg\min_{R} \sum_{i,j} F_{ij}\, m_{ij}(R), \qquad c_{ij}(R) = 1\big[z_i(R) \neq z_j(R)\big]. \tag{4}$$

After obtaining the optimal radii R^*, the geometric zone boundaries are formed not as circles, but as concave α-shape hulls of all stops within the corresponding radius. For each radius R_k, we compute an α-shape polygon of all stops with $d_i \leq R_k$. The geometric ring for zone k is obtained as the set difference between consecutive α-shape hulls. This yields ring-based zones that preserve the optimization logic of distance-based rings, while producing realistic, data-adapted geometries following the spatial distribution of stops rather than enforcing circular shapes.

To obtain a finite and comparable set of fare system candidates, we vary only the number of zones K. The weighting parameter $\lambda = 0.4$ is fixed based on exploratory sensitivity analyses showing that it provides a robust compromise between geographical proximity and flow-based connectivity; lower values overemphasize observed travel flows and produce irregular spatial partitions, while higher values lead to predominantly distance-driven clustering. For each selected K, the cell-based zoning model yields a unique assignment $Z^*(K)$ through agglomerative clustering on the combined distance metric. Although the objective combines geographical and flow-based separation costs, the resulting clustering solution serves as a deterministic heuristic rather than an exact optimizer. In the ring-based approach, the zoning problem reduces to a one-dimensional segmentation of stops by distance; for each K, the optimal radii $R^*(K)$ minimizing flow cuts are determined exactly, again resulting in a unique zoning configuration. Each zoning outcome directly defines one candidate fare system through the corresponding zone scheme. Thus, the complete candidate set consists of all zoning solutions obtained across the chosen values of K, for both the cell-based and ring-based approaches.

For each generated fare system, prices are assigned to all tariff steps, i.e., the fare levels associated with crossing a given number of zones. Passenger volumes are adjusted using a short-run fare elasticity of -0.3, a value frequently reported in the public transport literature as a representative estimate for urban bus demand (Holmgren, 2007). Based on these elasticity-adjusted volumes, tariff steps are optimized within a revenue-neutral framework: total revenue under the new fare system is required to match the revenue generated by the current baseline structure. The optimization problem maximizes total passenger volumes subject to this revenue constraint, ensuring that any increase in ridership is achieved without reducing overall revenue. Note that this revenue-neutrality constraint serves primarily as a calibration condition to make fare systems comparable under a common benchmark; alternative target levels, such as revenue increases or reductions, could

equally be imposed without changing the underlying optimization logic. Both Fixed-route and on-demand movements are included in the optimization, so that demand responses and revenue contributions from both subsystems jointly determine the resulting tariff steps. This setup ensures that differences between fare systems arise solely from their structural properties rather than from differences in total revenue.

Evaluation of Fare Systems

To evaluate the alternative fare systems, each scenario is assessed using two criteria: structural complexity and price-distance fairness. Structural complexity is operationalized as the *average number of zone crossings per passenger trip*, derived from the origin-destination (OD) passenger flow data. A lower number of zone crossing indicates a simpler and more stable segmentation.

Price-distance fairness is measured using the Gini index, computed from the ratio of paid fare to straight-line travel distance. For each passenger relation r, a distance-based fare ratio is defined as

$$f_r = \frac{fare_r}{d_r}, \tag{5}$$

which reflects how strongly the charged fare $fare_r$ deviates from strict proportionality to travel distance d_r. To quantify inequality across the network, the weighted Gini index is applied. Given fare-distance ratios f_r and passenger weights w_r, the weighted Gini index then follows as:

$$G = \frac{\sum_r \sum_{r'} w_r w_{r'} |f_r - f_{r'}|}{2(\sum_r w_r)(\sum_{r'} w_{r'} f_{r'})} \tag{6}$$

A lower Gini value indicates more distance-proportional pricing.

All fare systems are evaluated for both fixed-route and on-demand services, using average zone crossings and the Gini index of price-distance fairness. Since the two metrics differ in scale, zone-crossing values are rescaled such that the largest observed value equals one. The Gini index already lies within $[0, 1]$ by definition and therefore requires no further transformation. This normalization ensures comparability and allows both metrics to contribute meaningfully to subsequent multi-criteria evaluations.

Identification of Pareto-Optimal Fare Systems

To assess the overall performance, a combined performance point is defined for every scenario. For a fare system s, let $c_s^{FR}, c_s^{OD} \in [0, 1]$ denote the normalized zone-crossing values and $g_s^{FR}, g_s^{OD} \in [0, 1]$ the corresponding Gini indices for fixed-route (FR) and on-demand services (OD), respectively. A weighted combined point is defined as

$$x_s = \alpha \cdot c_s^{FR} + (1 - \alpha) c_s^{OD} \tag{7}$$

$$y_s = \alpha \cdot g_s^{FR} + (1 - \alpha) g_s^{OD}, \tag{8}$$

where $\alpha \in [0, 1]$ allow to set weight to the importance of fixed-route and on-demand services differently. The resulting set forms a two-dimensional evaluation space.

Subsequently, the Pareto front is identified and the relative desirability of each scenario is quantified by its distance to an ideal point. The ideal point does not represent a feasible fare system, it is a normative benchmark combining the best individual performance levels of each criterion and is defined as

$$x^* = \min_s x_s \tag{9}$$

$$y^* = \min_s y_s \tag{10}$$

In our case, $x^* = 0$ (no zone crossings in a flat fare system) and also $y^* = 0$ (ideal Gini index for completely distance-proportional tariff structure). For each scenario s, the (optionally weighted) Euclidean distance to this ideal solution is computed as

$$D_s = \sqrt{\gamma x_s^2 + (1 - \gamma) y_s^2}, \tag{11}$$

where $\gamma \in [0, 1]$ controls the relative emphasis on structural complexity versus price-distance fairness. This scalar measure provides a clear and interpretable ranking of both Pareto-efficent and dominant scenarios and supports the selection of integrated fare systems that best balance complexity and fairness across fixed-route and on-demand services.

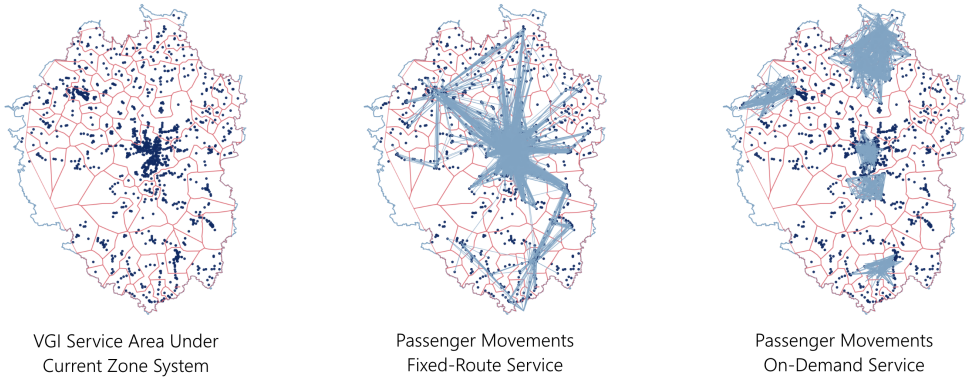


Figure 1: Current Zone System VGI Service Area and Passenger Movement Patterns

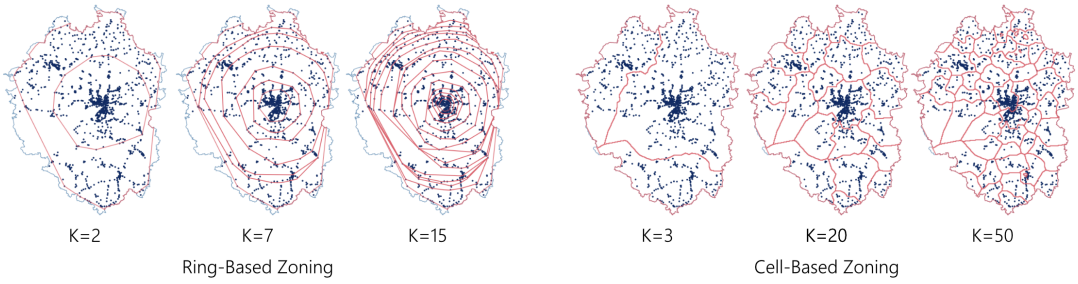


Figure 2: Spatial Partitioning of the Network under Alternative Zoning Schemes

3 Results and discussion

The analysis performed in this study is based on real-world data from the Ingolstadt region (Bavaria, Germany), where the regional transport association *Verkehrsverbund Großraum Ingolstadt* (VGI) operates a multi-modal network consisting of fixed-route bus lines and several on-demand services. One year of operational, ridership, and spatial data provides the empirical basis for OD movements and fares for both modes. Only single-ticket fares and their associated trips are considered.

The VGI applies a zone-based fare for fixed-route services composed of concentric rings that are further subdivided. As a result, many origin–destination pairs cross multiple boundaries even over short distances, generating sharp price discontinuities. Figure 1 illustrates the current zoning and movement patterns, highlighting numerous zone crossings.

On-demand services are integrated into the public transport network but follow a separate, tiered distance-based fare with 5km increments. These fares are considerably lower than those applied for comparable fixed-route trips, raising concerns regarding fare parity and potentially distorting demand between service types—effects that may reduce overall network efficiency. This makes the VGI system well suited to study how zonal granularity and zone-transition frequency influence the integration of on-demand services into a unified fare framework.

Overview of Constructed Fare Systems

To analyze the impact of spatial tariff structures, we generate zone-based systems with different spatial granularity. The cell-based approach includes 2–50 cells, while the ring-based approach comprises 2–15 concentric rings. Together with a flat-fare baseline, these systems span a design space from minimal structural complexity to highly differentiated zoning schemes aligned with mobility patterns (see examples in Figure 2).

Evaluation of Fare Systems

The changes in *average zone crossings* with increasing zonal granularity are shown in Figure 3, comparing ring-based and cell-based zoning. The status quo yields average of 0.28 zone crossings per trip, shown as the horizontal reference line.

The results show a clear pattern: In the *ring-based* system, all configurations up to seven rings produce fewer zone crossings than the status quo. Beyond this, the average number of crossings rises and eventually surpasses 0.28. In the *cell-based* system, the reduction is stronger. All configurations up to 30 cells remain below the status-quo level. Discontinuities (between 28–29 and 43–44 zones) occur when an added boundary lies on high-demand corridors.

Overall, both alternative zoning approaches demonstrate that a substantial reduction in fare-relevant boundary crossings is possible when the number of zones remains within moderate ranges. Beyond this threshold, crossings rise more steeply in ring-based systems, suggesting that, for the VGI network under consideration, either a cell-based system or a ring-based system with few rings is most effective for fixed-route services.

A comparison of the *price-distance fairness* (Gini Index) across zoning granularities is shown in figure 4. The status quo has a Gini coefficient of 0.37, indicating a relatively uneven distribution of zone sizes. Lower values indicate more distance-proportional fares and thus higher fairness.

Both approaches achieve lower inequality than the status quo once enough zones are introduced. In the *ring-based* systems, all configurations with four rings or more fall below 0.37. The *cell-based* system shows a similar pattern but at a different scale: configurations with twelve cells or more outperform the status quo, again increasing granularity yields a more proportional price-distance relationship.

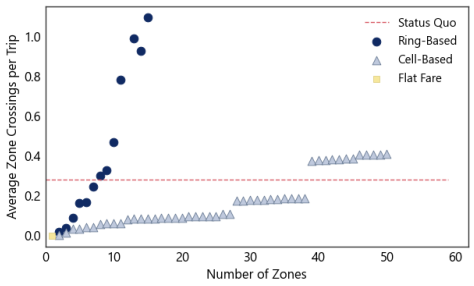


Figure 3: Average Zone Crossings per Trip (Fixed-Route Service)

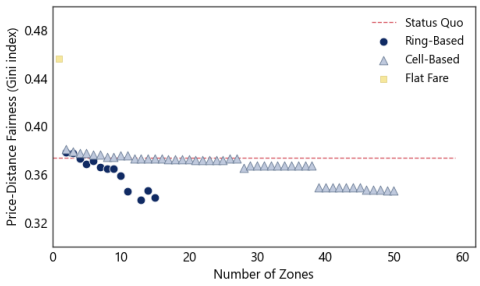


Figure 4: Price-Distance Fairness (Fixed-Route Service)

Identification of Pareto-Optimal Fare Systems

Figure 5 shows all generated fixed-route fare systems by structural complexity and price–distance fairness. The current fare system appears as a red reference marker. Several scenarios outperform the status quo with fewer zone crossings and a lower Gini index (ring-based systems with 4–7 rings and cell-based systems with 12–30 cells).

Figure 6 adds the corresponding on-demand performance of the selected systems. Average zone-crossing values are rescaled to $[0, 1]$. The results indicate that all cell-based fare systems and ring-based fare systems with up to six rings exhibit lower structural complexity on on-demand relations than the status-quo fare, but their fairness performance worsens relative to the currently applied tiered distance-based fare.

To evaluate system-wide performance, the fixed-route and on-demand outcomes are combined into a single performance point for each fare system. The gray markers in Figure 6 illustrate these combined values, with both metrics equally weighted. The Pareto-efficient fare structures are highlighted by red circular markers, indicating the non-dominated set across both criteria. Among these scenarios, the orange circular marker identifies the configuration with the minimum Euclidean distance to the ideal point, here corresponding to the 17-cell system.

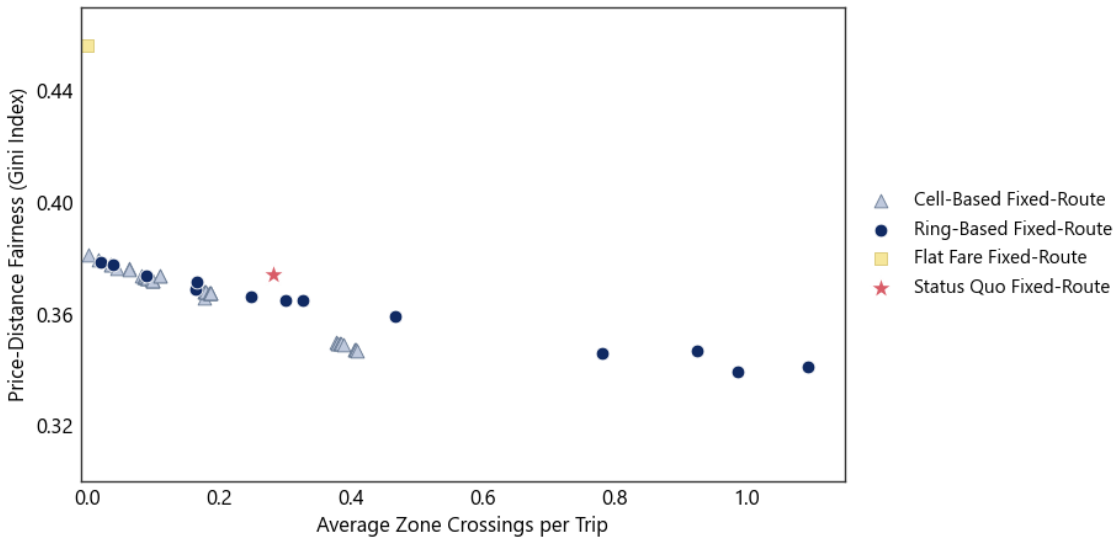


Figure 5: Structural Complexity vs. Fare Fairness for Fixed-Route Services

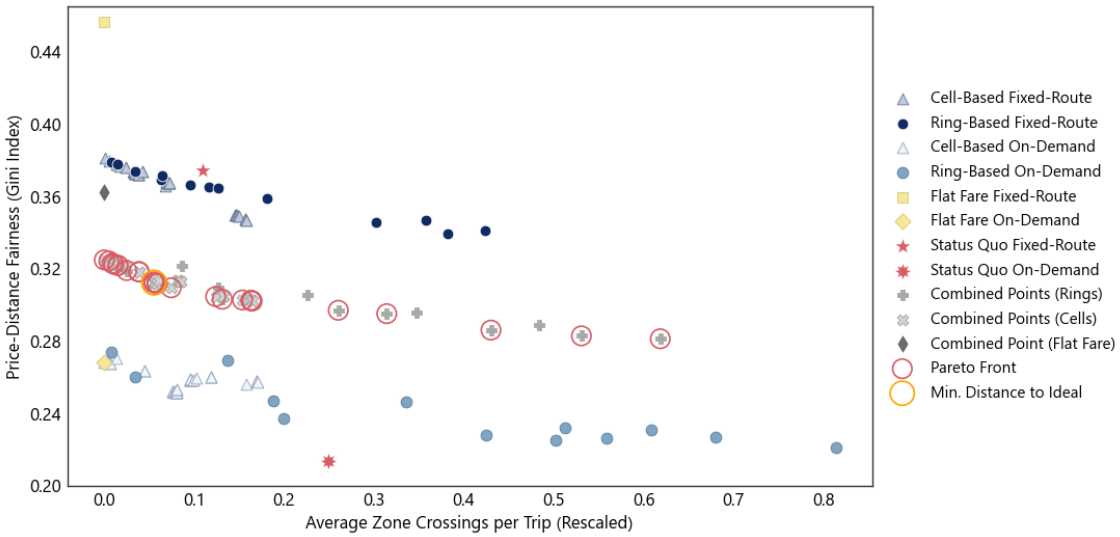


Figure 6: Integrated Assessment (weight=0.5) and Pareto-Efficient Fare Systems

4 Conclusions

This study develops a data-driven decision-support framework for designing integrated fare systems in multi-modal public transport networks. Using one year of operational, spatial, and ridership data from the Ingolstadt region, we generate and evaluate zoning-based fare systems for both fixed-route and on-demand services. The results demonstrate that the design of the zoning based on the movement pattern is an effective mechanism for reducing structural complexity of fare systems, as moderate levels of spatial granularity substantially lower the number of fare-relevant zone crossings.

By jointly assessing structural simplicity and price-distance fairness, and by extending each zoning scheme to the on-demand layer, the framework enables a coherent comparison of integrated fare systems. Through the construction of composite performance points, the identification of Pareto-efficient configurations, and the computation of distances to an ideal benchmark, the analysis reveals the most balanced and robust fare systems within a multi-criteria evaluation space.

Overall, the proposed framework provides practitioners and transport authorities with a transparent tool for selecting zoning configurations that support both operational simplicity and equitable fare outcomes across fixed-route and on-demand services, and thereby offering a structured pathway toward integrated fare systems in multi-modal public transport networks.

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