Embedding the Capabilities Approach in Transport Network Design: A Multi-Period Optimization Framework with Endogenous Feedback

Esta Qiu*[1], Aditya Paul[2], Michael Levin[3], Regine Gerike[1], and Travis Waller[1,4,5]

[1]"Friedrich List" Faculty of Transport and Traffic Sciences, TU Dresden, Dresden, Germany
[2]Intelligent Transport Systems, Transport Division, DTU Management, Technical University of Denmark
[3]Department of Civil, Environmental, and Geo-Engineering, University of Minnesota
[4]School of Civil and Environmental Engineering, UNSW Sydney, Sydney, Australia
[5]College of Engineering and Computer Science, ANU, Canberra, Australia

Short summary

This paper operationalizes the Capabilities Approach (CA) to inform resource allocation decisions in a transport network modeling framework. While existing studies recognize the role of the CA in equity-oriented policies and investments, integrating the normative theory into tractable analytical models remains limited. We address this gap by formulating a multi-period mixed-integer nonlinear network design problem that explicitly captures the relationships between resources, conversion efficiency, capability formation, and well-being. Conversion factors are represented through individual-specific travel costs and cost thresholds, enabling heterogeneous transformation of resources into capabilities. The framework further incorporates dynamic feedback, where achieved well-being lowers travel cost thresholds, thereby expanding spatial reach and influencing future capability formation. We compare feedback-aware and feedback-agnostic decision-makers, demonstrating how accounting for endogenous feedback alters optimal investment strategies. Additionally, we identify a critical capability threshold that provides an analytical interpretation of sufficiency-based justice. Sensitivity analysis highlights how population heterogeneity shapes long-term equity outcomes.

Keywords: capabilities approach, multi-period optimization, transport equity and justice, transport network design

1 Introduction

The Capabilities Approach (CA) (Sen, 1992; M. Nussbaum & Sen, 1993; M. C. Nussbaum, 2011) is a social justice theory that emphasizes the importance of personal, social and environmental factors, as well as individual freedom, in shaping well-being. In transport literature, the application of the CA framework often focuses on five core components, namely resources, conversion factors, capability set, choice and achieved functioning (Beyazit, 2011; Vecchio & Martens, 2021; Luz & Portugal, 2022). **Resources** are assets individuals can possess or access, most often defined as means of transport, but their usefulness depends on personal, environmental, and social **conversion factors**, which shape how effectively resources are transformed into capabilities. The **capability set** represents the set of possible **functionings**, or valued doings and beings, from which individuals can freely choose which functioning to achieve or realize. The achieved functionings then shape indivdiuals' **well-being**.

Despite growing conceptual advancements and efforts to operationalize the CA in the transportation context, few studies have explored how it can inform broader transport resource allocation decisions. Hananel & Berechman (2016) highlights the barriers to implementing the CA, which includes the setting of an appropriate minimum capability "threshold" (Pot et al., 2025; Ryan & Martens, 2023), the economic trade-offs against equity considerations (Barbero et al., 2024; Stokan et al., 2021) and political feasibility (Linovski et al., 2018; Rodríguez-Pose et al., 2018). However, these challenges are often discussed at a conceptual level, with limited analytical frameworks that embed them within tractable resource allocation models. Another key challenge of operationalizing the CA lies in quantifying the extent to which conversion factors affect individuals' ability to

transform resources into capabilities. Despite numerous studies (Rashid et al., 2010; Ryan et al., 2015; Ryseck & Behrens, 2024; Xu et al., 2024; Azmoodeh et al., 2023) acknowledging the influence of conversion factors on capability formation, the causal relationship through which these factors mediate the transformation of resources into capabilities remains largely unformulated.

This paper addresses these gaps by translating the normative concepts of the CA into quantifiable relationships within a network modelling framework. While most transport network models focus on system efficiency and travel costs, they rarely capture the mechanisms through which personal, social, and environmental conditions shape individuals' opportunities to participate in activities. Using a network modelling approach, this study advances the operationalization of the CA in transport resource allocation by embedding conversion factors, capability sets, and substantive freedom as computable relationships within the model. This enables the evaluation of infrastructure investments in terms of both efficiency and their implications for equity and individual opportunity.

The proposed framework bridges the technical rigor of network modelling with the normative depth of the CA, providing a novel approach for analyzing and designing transport systems through the lens of equity and freedom. More specifically, our contributions include the following:

1. We formulate a multi-period, mixed-integer nonlinear (MINLP) network design problem built upon the CA's theoretical relationships, formalizing the relationships between resources, conversion, capabilities, and well-being. This approach prioritizes resource allocation for disadvantaged users to bridge the gap with those who already possess high capability levels.
2. We address the challenge of identifying conversion efficiency by capturing the effects of conversion factors through personal travel costs and personal travel cost thresholds. These parameters define an individual's specific capacity to transform transport resources into actual location access.
3. In addition, we model the dynamism of capabilities, which is rarely quantified in transport literature, by portraying the feedback effect of well-being on personal growth or degradation over a multi-period setting.
4. We solve the multi-period MINLP and compare the performance of the feedback-aware and feedback-agnostic decision-makers. The feedback-agnostic decision-maker seeks to maximize marginal capability gains across the population without considering endogenous feedback effects. In contrast, the feedback-aware decision-maker incorporates these feedback mechanisms to target interventions that generate both direct gains and self-reinforcing improvements in future capability formation.
5. We investigate the relationship between the resultant resource allocation and individual capabilities. Based on this insight, we attempt to address the definitional challenge of the sufficiency threshold in CA through an analytical lens, by identifying a critical capability threshold C^*. Normatively, C^* represents the minimum capability level consistent with sufficiency-based justice. Mathematically, it is the critical capability value at which the feedback-sensitive resource allocation problem exhibits a structural shift, such that capability gains below and above this point have different implications for future capability formation.
6. As a sensitivity analysis, we generate multiple instances with varying levels of population heterogeneity, proxied by the variance in conversion efficiency, and compare the performance of the two decision-makers. The results are used to characterize how heterogeneity interacts with feedback mechanisms to shape long-term investment trajectories.

2 Model definition

Modelling the CA relationships from a network perspective

Consider a transport network with a set of nodes N and a set of links L. Consider further that individuals p of a population P reside in the network and carry out functions at nodes in the network. The nodes which can serve as function destinations belong to a subset $J \subset N$ of the network nodes. Trip costs are non-uniform across the population. While a conventional link travel cost c_l exists for every link l, each individual p experiences a personalized link travel cost. This is determined by a personalization factor γ_l^p which accounts for individual-specific barriers or

disadvantages, such as socio-economic status, physical mobility, or vehicle access. Let $x_{lj}^p \in \{0,1\}$ be the routing decision variable, where $x_{lj}^p = 1$ if link l belongs to the shortest path for user p traveling from their origin node o_p to node j. Their personalized trip cost $C_{o_pj}^p$ is the summation of these personalized costs along the optimal path:

$$C_{o_pj}^p = \sum_{l \in L} (c_l \cdot \gamma_l^p) \cdot x_{lj}^p \tag{1}$$

Resources and conversion factors

In the context of a transport infrastructure planning problem, the resources would be elements of the transport infrastructure, and the conversion factors would be the individualized circumstances that affect a person's ability to use this infrastructure to reach activity destinations.

Individual access is governed by a personalized trip cost threshold T_p, representing the maximum resource expenditure (in time, money, or effort) an individual can sustain for a single trip. A person can only reach node j from node i if the personalized trip cost C_{ij}^p does not exceed T_p. Let α_{ij}^p denote if person p can reach node j from node i.

$$\alpha_{o_pj}^p = \begin{cases} 1 & \text{if} \quad C_{o_pj}^p \leq T_p \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Every destination j harbors a set of functions categorized by type k (e.g., medical, educational, consumer), each possessing a specific quality index z. Let o_p be the residential node for person p, and σ_j^{kz} be the number of type k functions with quality z at node j. The number of type k functions of quality z reachable by person p is given by:

$$A_{kz}^p = \sum_{j \in N} \sigma_j^{kz} \cdot \alpha_{o_pj}^p \qquad \forall p \in P,\ \forall k \in K,\ \forall z \in Z \tag{3}$$

Using generalized travel cost as a proxy for the effort, time, and monetary expense required to reach destinations, population heterogeneity is reflected in two distinct components: (i) the personalized cost of utilizing a specific transport infrastructure, and (ii) the personalized cost threshold that determines whether a trip is viable for an individual. This threshold is defined not by willingness to pay, but by whether a trip's cost preserves an individual's substantive freedom i.e., it must not render other valued functionings within their capability set infeasible (Sen, 1992). A location is included in an individual's choice set if and only if its personalized travel cost falls below their personal viability threshold. This criterion operationalizes how conversion factors transform transport resources into genuine travel capabilities.

Within this modelling framework, the user-group conversion efficiency ϕ_p is back-calculated as the ratio of the accessible choice set (capability) to the total number of activity locations $|D|$ theoretically available in the system. This conversion factor serves as a high-level indicator of how effectively a person can translate the objective existence of resources into tangible accessibility, essentially capturing the systemic and personal interactions in the allocation process.

$$\phi_p^t = \frac{\sum_{k \in K} \sum_{z \in Z} A_{kz}^{pt}}{|D|} \tag{4}$$

Capabilities and functionings

Anand et al. (2005) follows Sen's definition of capabilities as the feasible alternative combinations of functionings, and defines the capabilities vector $\mathbf{C}$ as an n-dimensional measure that gives rise to a scalar value. In many transport studies, capabilities are understood as an individual's potential to access and engage with opportunities, while functionings reflect the actual behaviors or outcomes observed such as activity participation (Bantis & Haworth, 2020). As such, the accessibility-capability vector may represent the set of activity locations an individual can access to achieve valued functionings. This is similar to the concept of cumulative opportunities which is one of the most common accessibility measures in equity analysis (Li et al., 2023).

The CA accounts for human diversity by focusing on the heterogeneous conversion factors and

the plurality of functionings and capabilities (Robeyns, 2005). As some dimensions may be more important for some groups but less so for others, well-being evaluations should consider a wide range of dimensions, as well as the quantity and quality of functionings and capabilities (Robeyns, 2017). Building on this foundation, the capability of an individual is defined by the quantity, quality and diversity of activity locations found within their choice set. Let q_{kz} be the corresponding quality score of quality index z for a function k, then the quality-weighted choice set for person p is given by:

$$\bar{A}_k^p = \sum_{z \in Z} A_{kz}^p \cdot q_{kz} = \sum_{j \in N} \alpha_{oj}^p \cdot \sum_{z \in Z} \sigma_j^{kz} \cdot q_{kz} \qquad \forall p \in P,\ \forall k \in K \tag{5}$$

Quality scores $q_{k,z}$ are scaled within function-specific ranges $[q_k^{min}, q_k^{max}]$ to capture two distinct priorities. First, in the absence of accessible functions, the capability increase obtained by first access to a function is different across functions. Second, once basic capability is established, marginal capability gains from quality upgrades differ across functions. For example, when moving to a new city, access to a grocery store is essential. But once established in that city, access to better hospitals is more important than access to better groceries.

The objective function computes individual capability Cap^p by leveraging the maximum quality score q_k^{max} to scale long-term improvements while preserving initial access sequences using the entry level lowest quality score q_k^{min}:

$$Cap^p \leq \sum_{k \in K} q_k^{max} \cdot \ln\left(\overline{A}_k^p \cdot \frac{q_k^{min}}{q_k^{max}} + 1\right) \tag{6}$$

This ensures that capability only increases significantly when an individual gains access to a diverse array of high-quality resources, rather than a surplus of a single resource type.

Choice

From a planning perspective, the primary public responsibility is to remove constraints on freedom i.e., expand capabilities. We follow Sen's distinction between well-being achievement (chosen functionings) and well-being freedom (the capability set), which Sen argues has intrinsic value regardless of whether individuals exercise all options. Once genuine options exist, choice becomes a matter of personal autonomy. As such, our proposed model does not model individual choices, but focuses on expanding the freedom itself.

Temporal Capability Feedback Mechanism

The personalized travel cost represents the burden of a single trip, primarily determined by relatively stable mobility-related conversion factors and external infrastructure. The personalized cost threshold represents the individual's total capacity to absorb burdens without deprivation i.e., a viability constraint. In the proposed model, improvements in overall well-being (from expanded capabilities) are assumed to increase an individual's personalized cost threshold. An increase in well-being through improved health, stronger social support etc. expands an individual's overall capacity to bear costs, thereby relaxing this viability constraint and enlarging the set of genuinely accessible destinations. This approach captures how gains in one domain of life may directly expand an individual's substantive freedom, without conflating this capability growth with changes to the transport system itself.

The proposed model incorporates a dynamic feedback loop to capture the long-term evolution of individual capability. We posit that an increase in an individual's capability metric leads to a delayed expansion of their personalized trip threshold T_p. As an individual accesses better health, education, or economic opportunities, their capacity to absorb travel costs increases. This is modeled as a sigmoid function:

$$T_p^{t+1} = T_{min} + \frac{T_{max} - T_{min}}{1 + e^{-\theta(Cap_p^t - C^*)}} \tag{7}$$

In the context of Sen's Capabilities Approach, C^* provides a formal definition of "basic functions". We define the prescribed capability minimum (C^*) endogenously as the inflection point of the entropy-based capability function. This point marks the transition from essential diversity acquisition to functional redundancy. Mathematically, C^* is the point where the marginal gain of

capability relative to the travel budget stops accelerating and starts slowing down i.e., the second derivative of the capability function Cap with respect to the travel cost threshold T:

$$\frac{d^2Cap}{dT^2} = 0 \tag{8}$$

At the convex phase ($\frac{d^2Cap}{dT^2} > 0$) or low levels of capability, small increases in T lead to rapid gains in diversity as they encounter their first instances of essential activities. At the concave phase ($\frac{d^2Cap}{dT^2} < 0$), the individual has reached functional saturation, such that further network expansion leads to diminishing returns in entropy.

Network Design

In order to improve the transport infrastructure, the network manager has a fixed budget B^t in each time period. This budget is spent in adding links to the network, with the intention of increasing accessibility to function destinations. ψ_B indicates the per unit length cost of building a new link (of unit capacity). Under this framework modifications to network topology are encapsulated by

$$\sum_{l \in L_c} (\psi_B \cdot d_l \cdot y_l) \leq B^t \tag{9}$$

3 Multi-period Capability-based Network Design Problem

In a multi-period setting, the investment sequence has important implications because early interventions generate benefits that accrue across subsequent periods. The presence of feedback effects introduces an additional layer of temporal dependence, as early investments can alter the marginal productivity of future allocations by improving the ability of individuals to convert resources into capabilities. Formulation 10 shows the multi-period, feedback-aware network design problem.

$$\max\ Z = \sum_{t \in T} \sum_{p \in P} Cap_p^t \lambda_p \tag{10a}$$

$$C_{o_pj}^{pt} = \sum_{l \in L} c_l \cdot \gamma_l^p \cdot x_{lj}^{pt} \qquad \forall p \in P, j \in J, t \in T \tag{10b}$$

$$C_{o_pj}^{pt} \leq T_p^t + M(1 - \alpha_{o_pj}^{pt}) \qquad \forall p \in P, \forall j \in J, \forall t \in T \tag{10c}$$

$$C_{o_pj}^{pt} \geq T_p^t - M(\alpha_{o_pj}^{pt}) \qquad \forall p \in P, \forall j \in J, \forall t \in T \tag{10d}$$

$$\bar{A}_k^{pt} \leq \sum_{j \in N} \alpha_{oj}^{pt} \cdot \sum_z \sigma_j^{kz} \cdot q_p^{kz} \qquad \forall p \in P, \forall k \in K, \forall t \in T \tag{10e}$$

$$Cap_p^t \leq \sum_{k \in K} q_k^{max} \cdot \ln\left(\overline{A}_k^p \cdot \frac{q_k^{min}}{q_k^{max}} + 1\right) \qquad \forall p \in P, \forall t \in T \tag{10f}$$

$$\sum_{l \in L_{new}} \psi_B \cdot d_l \cdot y_l^t \leq B^t \qquad \forall t \in T \tag{10g}$$

$$T_p^{t+1} \leq T_{min} + \frac{T_{max} - T_{min}}{1 + e^{-\theta(Cap_p^t - C^*)}} \qquad \forall p \in P, \forall t \in T \backslash \{|T|\} \tag{10h}$$

$$\sum_{l \in \Gamma^+(n)} x_{lj}^{pt} - \sum_{l \in \Gamma^-(n)} x_{lj}^{pt} = d_{o_pj} \qquad \forall n \in N, \forall j \in J, \forall p \in P, \forall t \in T \tag{10i}$$

$$x_{lj}^{pt} \leq y_l^t \qquad \forall l \in L_{new}, \forall j \in J, \forall p \in P, \forall t \in T \tag{10j}$$

$$y_l^t \in \{0, 1\} \qquad \forall l \in L_{new}, \forall t \in T \tag{10k}$$

$$\alpha_{o_pj}^{pt} \in \{0, 1\} \qquad \forall p \in P, \forall j \in J, \forall t \in T \tag{10l}$$

$$x_{lj}^{pt} \in [0, 1] \qquad \forall j \in J, \forall l \in L, \forall p \in P, \forall t \in T \tag{10m}$$

Feedback-agnostic and **feedback-aware** decision-makers maximize total capability ($\sum Cap_p^t$) over the planning horizon, allocating resources to maximize marginal gains across a broad population. λ_p represents the population of each subgroup $p \in P$, and capability is defined in Equation

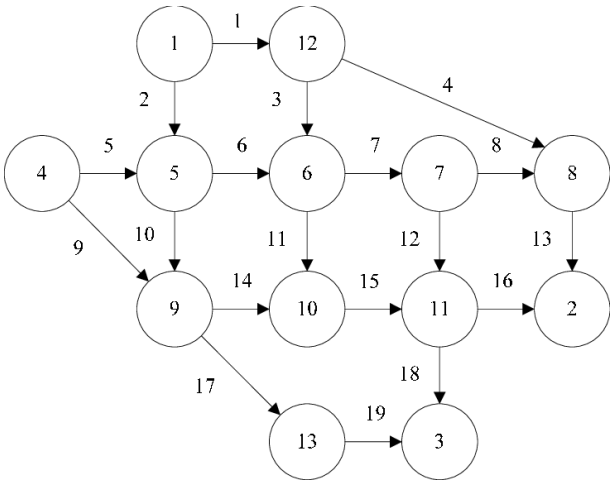


Figure 1: Nguyen Dupuis Network

Link attributes
Motorized only, no transit
Tolled road
No step-free access to transit stop
Low security / high traffic stress
Complex interchange
Transit only link

Table 1: Link attributes

10f. Personalized travel cost for each user groups is defined in Equation 10b. Equations 10c and 10d identify the accessible nodes based on their personalized travel cost threshold. Equation 10e computes the quality-weighted score of an individual's choice set, which contributes to their capability. Equation 10g enforces the design budget, and L_{new} is the set of candidate links for investment. Equation 10i is the flow conservation constraint. Equation 10j ensures that only built links can be included in the shortest path.

The feedback-agnostic decision-maker solves the model without Equation 10h, thus neglecting the feedback effect at the decision stage, even though the feedback mechanism operates in reality. The resulting allocation is subsequently evaluated ex post under the full feedback specification to obtain the corresponding realized capability levels. On the other hand, the feedback-aware decision-maker solves the model with Equation 10h, internalizing the feedback mechanism within the optimization. As a result, resource allocation reflects both immediate capability gains and their dynamic propagation over time.

Experimental setup

Consider the ND network (Figure 1) with 13 nodes, 19 links, and 4 OD pairs. We assume node 1 and node 4 to be residential nodes, node 2 and node 3 are destination nodes with different activity locations. The link attributes (Table 1), and personal attributes assigned to each user group (Table 2) embed the effects of environmental, personal and social conversion factors.

Each link attribute affects users differently based on their attributes, represented through attribute-specific lower and upper bounds. Within these bounds, the realized effect is sampled from a truncated normal distribution, allowing for heterogeneity while ensuring that values remain within the specified range. Each link may be assigned multiple non-conflicting link attributes, and each user may possess multiple non-conflicting attributes. Aggregation rules are then applied to derive link-specific multipliers for each user.

Category of conversion factor effect	Attribute associated with the user group
Environmental	Car-dependent
Personal	No access to car
Social	Vulnerable population
Social	Low income
Personal	Mobility-impaired
Personal	Unfamiliar with Transport / ICT
Environmental	Live in node 4 (lower-quality public transport infrastructure)

Table 2: Agent attributes (conversion factors)

Solution method

We use outer-approximation to linearize the non-linear objective function. We first reformulate the MINLP (10) into a path-based formulation with linearized objective, and then use column generation to iteratively populate the Restricted Master Problem (RMP). In this process, the dual prices derived from the RMP are used to identify and include new improving paths; specifically, these duals inform the pricing subproblem, which generates additional path variables to be added to the basis until an optimal solution is reached. We also develop a branch-and-price algorithm to solve the path-based model efficiently, as this approach is particularly well-suited for optimization problems characterized by a vast number of variables but relatively sparse constraints.

4 Conclusion

This paper bridges normative justice theory and technical network modelling by operationalizing the Capability Approach and analytically addressing key challenges in translating its principles into tractable resource allocation models. Numerical experiments are currently in progress, results will be presented at the conference.

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