Strategic game between on-demand operators with incomplete demand information

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SHORT SUMMARY

This study investigates the strategic interactions between competing transport network companies (TNCs) within a multi-modal transportation system at both operational and informational levels. A bi-level model is developed, where at the upper level, each TNC independently optimizes its trip fare to maximize profit, and at the lower level, travelers select among available services according to a multinomial logit (MNL) model. To better capture real-world uncertainty, the model incorporates an incomplete-information setting, where TNCs do not have exact knowledge of travelers' value of in-vehicle time (VTT), value of waiting time (VWT), and the total demand. Instead, they must learn and iteratively update their beliefs based on observed outcomes, following a Bayesian framework. Numerical experiments demonstrate that the market size plays a decisive role in the accuracy of information inference. The benefits of ignorance are also shown in our results, that less information may help to mitigate the intensity of competition.
Keywords: bi-level optimization, incomplete information, generalized Bayesian Nash equilibrium

1 INTRODUCTION

The liberalization of the mobility market with recent advancements in information technology development has given rise to various mobility services, such as ride-hailing and ride-sharing. These emerging service providers greatly enrich options for travelers but also complicate the urban transportation system at the operational level as their services could be both substitutive and complementary to each other. Meanwhile, the service providers often have contradictory goals while interacting with various operational strategies subject to common limited resources (e.g., travelers, drivers, road capacities, etc.). Therefore, the competition among different transport service providers (TSPs) with conflicting objectives has been a heated topic in both research and practice (e.g., Ke et al., 2021; Ni et al., 2021; Li et al., 2021).

Research on competition in mobility markets generally falls into two categories: competition among TSPs offering similar services and competition among different transport modes. J. Zhou et al. (2005) propose a bi-level transit equilibrium model with profit-maximizing operators adjusting prices at the upper level and passengers choosing options based on service quality and fares at the lower level, while Feng et al. (2016) focus on operators optimizing headways instead of prices. Studies such as Zhang & Nie (2021a) and Cai et al. (2024) model the duopoly competition between ride-hailing platforms, both of which consider platforms compete for limited demand (passengers) and supply (drivers). Specifically, the former examines optimal pricing under different scenarios, including pooling availability, driver exclusivity, and regulatory constraints. The latter extends the static equilibrium model into an evolutionary process capturing varying market conditions, while introducing a service aggregator and a mixed fleet of gasoline and electric vehicles. Extended to more complex markets with multiple players, Y. Zhou et al. (2022) model the ride-sourcing market with multiple competing platforms, considering scenarios with and without a service aggregator. Recently, Huang et al. (2024) propose a generalized multi-modal equilibrium model in which various TSPs coexist within the same network and compete by setting link fares, subject to subsidies from the regulator. A major shortcoming in existing research on competition within mobility systems is the implicit assumption of complete information. In reality, however, competition typically unfolds under incomplete information, where TSPs cannot make perfectly informed decisions, and the information available to them is often asymmetric and nonexclusive. This makes the information structure more complex, yet also more compelling to investigate.

Research on games with incomplete information largely builds on the seminal work of Harsanyi (1967), which models information uncertainty through Bayesian games. In this framework, players hold prior

beliefs over unknown parameters and update them using observed signals via Bayes' rule, adjusting strategies accordingly until reaching a Bayesian Nash equilibrium. Extending the Bayesian game framework, Chriss (2025) analyzes strategic trading in competitive markets, where firms lack complete information about competitors' target trade sizes and market impact models. Incorporating belief heterogeneity, they derive optimal strategies and show substantial behavioral shifts relative to the full-information benchmark. In the context of transportation, Q. Liu & Chow (2022) model multiple public transit (PT) operators with incomplete information about the travel demand and user preferences. Accordingly, the frequencies are optimized by each operator based on their own observations of the market and common knowledge of travelers. J. Liu et al. (2016) use a Bayesian congestion game to study the route choice of users with heterogeneous information, finding that users without access to information benefit from informed ones, and additional high-accuracy data has little effect once most users are well-informed.

This work aims to model interactions within the mobility system with a particular focus on the competition among transport network companies (TNCs). To bridge the gap between analytical models and real-world practices, incomplete demand information is considered in the setting. Particularly, this work aims to answer the following questions:

- How can TNCs approximate the state of the entire market based on observations from their operations?
- How do the beliefs of TNCs evolve with newly collected observations, and under what conditions can these beliefs converge to the true market settings?
- Given these imperfect and evolving beliefs, how do TNCs design and optimize pricing strategies?

2 MODEL FORMULATION

Three groups of stakeholders are considered in this study, namely, transport network companies (TNCs), the public transit (PT) operator, and travelers. Their decision-making problems and interactions are characterized in a bi-level structure. At the upper level, TNCs decide on their trip fares. Since there is more than one TNC in the market, each TNC's decision is affected by others. For simplicity, we assume that PT implements a fixed unit fare and has sufficiently large capacity to serve all demand, which is also common knowledge in the market. Both TNCs have fixed fleet sizes, and their service capacities are expressed by total vehicle distances. At the lower level, travelers choose among trips offered by TNCs or PT, with the objective of maximizing their own travel utilities.

Lower level: Market equilibrium based on aggregate demand model

We adopt a multinomial logit model (MNL) to model the mode choice of travelers based on the trip fare and total travel time. We classify travelers into a finite number of user classes, denoted as a set I. For travelers of class $i \in I$, the generalized cost of service m is specified as follows:

$$U_{im} = b_m + f_m(\ell_i) + \beta_i^t t_m(\ell_i) + \beta_i^w w_m(\ell_i), \tag{1}$$

where b_m is the service-specific disutility measured in monetary cost, ℓ_i denotes class-specific average direct trip distance. $f_m(\cdot), t_m(\cdot), w_m(\cdot)$ are service-specific functions for trip fare, in-vehicle time and waiting/access/transfer time, and β_i^t, β_i^w are class-specific values of in-vehicle time and waiting/access/transfer time.

The probability that class i choosing service m is computed as

$$P_{im} = \frac{\exp(-\theta U_{im})}{\sum_{m' \in \mathscr{M}} \exp(-\theta U_{im'})}, \quad \forall m \in \mathscr{M} \tag{2}$$

where $\mathscr{M} = \{j\}_{j \in J} \cup \{P\}$ denotes the joint set of all TNCs and PT.

Let D_i be the total number of travelers in class i, with the average direct trip distance ℓ_i, then the total demand of class i expressed in travel distance for service m is computed as:

$$Q_{im} = D_i \ell_i P_{im}, \tag{3}$$

and the total demand for service m is

$$Q_m = \sum_{i \in I} Q_{im}, \tag{4}$$

In what follows, we specify each element in the generalized cost Eq. (1). Following the real-world practice, we assume the TNC trip fare is distance-based, while the PT fare is segment-based. Let f_m be the unit price of service m, then the trip fare for travelers with trip distance ℓ_i is given by

$$\text{TNC:} \quad f_{T_j}(\ell_i) = f_j \delta_T \ell_i \tag{5}$$

$$\text{PT:} \quad f_P(\ell_i) = f_P(\alpha \ell_i + 1) \tag{6}$$

where δ_T is the road-network detour factor reflecting the ratio of vehicle travel distance to the line distance, α is an exogenous parameter that gives the average number of transfers per unit line distance.

The in-vehicle time for TNC and PT is computed with service-specific speed v_m. The formulas of trip time for travelers with trip distance ℓ_i are as follows:

$$\text{TNC:} \quad t_{T_j}(\ell_i) = \frac{\delta_T \ell_i}{v_T} \tag{7}$$

$$\text{PT:} \quad t_P(\ell_i) = \frac{\delta_P \ell_i}{v_P} \tag{8}$$

where δ_P is similarly defined as the transit-network detour ratio.

For TNC waiting time, we adopt the general form proposed in (Y. Zhou et al., 2022):

$$w_{T_j} = A(N^v_{T_j})^{-k}, \tag{9}$$

where A counts the exogenous factors in the matching process, $N^v_{T_j}$ is the number of vacant vehicles, and $k \in (0,1]$ is the sensitivity parameter and set to be 0.5 for regular e-hailing service without passenger competition (Zhang et al., 2025).

Finally, the total access, egress, and transfer time for PT is computed as:

$$w_P(\ell_i) = 2\tau_a + \tau_t \alpha \ell_i, \tag{10}$$

where τ_a refers to the access and egress time, and τ_t is the time for each transfer.

The travel demand specified in Eq. (4) is endogenously determined by the market equilibrium because passenger waiting time for TNC services is determined by the demand-supply relationship described by the flow conservation constraints:

$$C_j = N^v_j \ell_0 + Q_j, \quad \forall j \in J \tag{11}$$

where C_j denotes the total service capacity of TNC j, ℓ_0 is the average vehicle travel distance per unit time.

It can be easily verified that Eq. (11), along with the demand functions Eq. (3)-(4), form a fixed point $x = F(x)$ with $x = (w_j(\ell_i))_{j\in J, i\in I}$. In other words, the fixed-point solution corresponds to the market equilibrium where no traveler has incentives to further change their mode choices. The existence of equilibrium can also be proved by evoking Brouwer's fixed-point theorem (Brouwer, 1911).

Upper level: Equilibrium between TNCs

We characterize the uncertain market condition through three sets of state variables: demand scale ($D = (D_i)_{i\in I}$), travelers' value of in-vehicle time ($V = (\beta^t_i)_{i\in I}$), and travelers' value of waiting time ($W = (\beta^w_i)_{i\in I}$). A full market state ω is then defined as a combination of these state variables: $\omega = (D, V, W) \in \Omega$, where Ω is the finite state space. We assume the state variables each follow a discrete distribution over three values: high (H), medium (M), and low (L), and they are fully independent of each other. Accordingly, the joint distribution of full market state is discrete and has $3^{3|I|}$ possible realizations.

Let $\pi^j(\omega)$ denote the prior belief of TNC $j \in J$ about the probability of state $\omega \in \Omega$, and $\mathscr{O}$ express the collective observations from the market during a unit period of operations (e.g., a single day). The belief is updated using Bayes' rule:

$$\pi_j(\omega \mid \mathscr{O}) = \frac{p_j(\mathscr{O} \mid \omega)\pi_j(\omega)}{\sum_{\omega' \in \Omega} p_j(\mathscr{O} \mid \omega')\pi_j(\omega')}, \tag{12}$$

where $p_j(\mathscr{O} \mid \omega)$ is the probability of observation $\mathscr{O}$ given market state ω.

In what follows, we specify $P_j(\mathscr{O} \mid \omega)$ as the probability of a particular realization of demand Q_j under market state ω. Let n_{ij} be the number of travelers in class i choosing TNC j, then $Q_j = \sum_{i \in I} n_{ij} \ell_i$. Since the individual mode choices are independent, we have $n_{ij} \sim \text{Bionomial}(D_i(\omega), P_{ij}(\omega))$, where $D_i(\omega)$ and $P_{ij}(\omega)$ denotes the total demand and choice probability of traveler class i in market state ω, respectively. Accordingly, the joint probability is given by

$$p_j(\mathscr{O} \mid \omega) = \prod_{i \in I} \binom{D_i(\omega)}{n_{ij}} P_{ij}(\omega)^{n_{ij}} \big(1 - P_{ij}(\omega)\big)^{D_i(\omega) - n_{ij}}. \tag{13}$$

When the demand D_i is sufficiently large for any class i, Eq. (13) can be approximated by a normal distribution according to the central limit theorem, which yields

$$p_j(\mathscr{O} \mid \omega) \approx \frac{1}{\sqrt{2\pi\sigma_j^2(\omega)}} \exp\left(-\frac{\big(Q_j - \mu_j(\omega)\big)^2}{2\sigma_j^2(\omega)} \right), \tag{14}$$

where $\mu_j(\omega) = \sum_{i \in I} D_i(\omega) \ell_i P_{ij}(\omega)$, and $\sigma_j^2(\omega) = \sum_{i \in I} D_i(\omega) \ell_i^2 P_{ij}(\omega) \big(1 - P_{ij}(\omega)\big)$.

Given the belief about the market condition, denoted as $\pi_j(\omega)$ for simplicity, each TNC j determines its trip fare rate per unit distance f_j to maximize its own profit while competing with the other TNCs. The optimal pricing problem is formulated as:

$$\begin{aligned}
\max_{f_j} \quad & E_\omega \left[f_j \delta_T Q_j(f_j; f_{-j} \mid \omega) - u_j C_j \right] && \text{(15a)} \\
& = \sum_{\omega \in \Omega} \left[f_j \delta_T Q_j(f_j; f_{-j} \mid \omega) - u_j C_j \right] \pi_j(\omega) \\
s.t. \quad & Q_j(f_j; f_{-j} \mid \omega) \le C_j \quad \forall \omega \in \Omega, && \text{(15b)} \\
& 0 \le f_j \le K_T && \text{(15c)}
\end{aligned}$$

where C_j denotes the service capacity measured by distance, u_j is the unit operating cost, and K_T denotes a sufficiently large constant as the upper bound of feasible trip fares. Objective (15a) is computed as the total revenue of trip fare $f_j \delta_T Q_j$ minus the total operating cost $u_j C_j$, Constraint (15b) claims that the service capacity must be sufficient to serve TNC trips, and Constraint (15c) defines the feasible trip fare.

Equilibrium and solution method

Let $f = (f_j)_{j \in J}$ denote the joint pricing strategy of all TNCs and f_{-j} denote the pricing strategy of TNCs other than TNC j. We further introduce Θ_j to denote the feasible set of f_j given by constraint (15c), and let $\Theta = \cup_{j \in J} \Theta_j$ to represent the joint feasible set of f without coupled constraint (15b).

The outcome of the upper-level equilibrium between TNCs is characterized as a Generalized Bayesian Nash Equilibrium (GBNE). Specifically, we focus on a subset of equilibria that satisfies the stochastic variational inequality (SVI) conditions (Franci & Grammatico, 2022):

$$\langle -G(f^*), f - f^* \rangle \ge 0, \quad \forall f \in \Gamma(f^*) \tag{16}$$

where $G(f) := \text{col}(G_j(f))_{j \in J}$ is the collective expected pseudogradient mapping. Each component is the gradient of expected profit according to TNC j's private belief π_j:

$$G_j(f) = \nabla_{f_j} E_\omega \left[f_j \delta_T Q_j(f_j, f_{-j} \mid \omega) - u_j C_j \right], \tag{17}$$

and $\Gamma(f^*) = \{ f \in \Theta \mid Q_j^*(f_j^*; f_{-j}^* \mid \omega) \le C_j, \forall j \in J, \omega \in \Omega \}$ denotes the coupled joint feasible region.

To facilitate the solving procedure, we rewrite the Lagrangian of the equivalent minimization problem that relaxes the capacity constraint (15b):

$$\mathscr{L}_j(f_j, \lambda_j) = \sum_{\omega \in \Omega} \left[-f_j Q_j(f_j, x^*; f_{-j} \mid \omega) + u_j C_j + \lambda_j(\omega) \left(Q_j(f_j, x^*; f_{-j} \mid \omega) - C_j \right) \right] \pi_j(\omega), \tag{18}$$

where $\lambda_j(\omega)$ is the Lagrangian multiplier associated with the market state ω.

In each iteration, we first solve the lower-level market equilibrium $x^*(f)$ at the current pricing strategy f, then evaluate the equilibrium sensitivity $\partial x^*/\partial f$, and finally integrate them into the projected gradient descent step to update the upper-level pricing f and multipliers λ. Specifically, we follow the basic differential multiplier method (BDMM) to simultaneously update f and $\lambda = (\lambda_j(\omega))_{\omega\in\Omega, j\in J}$ (Platt & Barr, 1987). Specifically, the equilibrium sensitivity is computed using the implicit function theorem following previous studies (e.g. Zhang & Nie, 2021b) as follows:

$$\frac{\partial x^*}{\partial f} = \nabla_f F(x^*, f) + \nabla_x F(x^*, f)\frac{\partial x^*}{\partial f}, \tag{19}$$

The detailed algorithm would be provided in full paper.

3 RESULTS AND DISCUSSION

We conduct numerical experiments to investigate the equilibrium outcomes, along with the strategies and beliefs of TNCs, under different market settings. We begin with a baseline scenario with homogeneous travelers, and then extend the analysis to settings of heterogeneous travelers, small market scale, and incorrect prior knowledge.

Baseline scenario

The baseline scenario contains a single traveler class, and the possible values of state values are reported in Table 1, along with the TNC capacities. Specifically, the total capacity is below the high-demand state, indicating a less competitive market, whereas it is larger than the low-demand state, where more intense competition is expected.

State Level	**Low**	**Medium**	**High**
Demand (km)	30,000	50,000	80,000
VTT (CHF/h)	3	6	10
VWT (CHF/h)	5	10	15
Capacity of TNC 1 (km)		24,000	
Capacity of TNC 2 (km)		32,000	

Table 1: Baseline settings.

True state	L-L-M		M-L-M		H-L-M	
	TNC1	TNC2	TNC1	TNC2	TNC1	TNC2
Price (CHF)	0.168	0.182	0.310	0.340	0.581	0.606
Profits (CHF)	386	871	5574	8608	11559	16527
Market share (%)	42.5	57.4	39.5	53.4	23.1	31.3
Final belief (prob.)	L-L-M(1)	L-L-M(1)	M-L-M(1)	M-L-M(1)	H-L-M(1)	H-L-M(1)

Table 2: Results under baseline setting.

Results of the baseline experiment are reported in Table 2 with varying demand levels and fixed values of time. Specifically, the true state is expressed as the level of demand, VTT, and VWT (e.g., L-L-M means low-demand, low-VTT, and medium VWT). In all tested cases, both TNCs are able to estimate the correct market state and the equilibrium pricing strategy increases with the demand level. Besides, TNC2, with a larger service capacity, always enjoys a larger market share and a higher profit.

The correct estimation of market condition is possibly attributed to the homogeneous travelers and the large market size. Hence, in the following experiments, we examine the robustness of the results towards heterogeneous travelers and smaller market sizes.

Heterogeneous travelers

In the second experiment, we consider two traveler classes with different trip distances and values of time. While the former can be observed from realized trips, the latter is unknown to TNCs. The state values and

other parameters are reported in Table 3. Note that "Traveler ratio" refers to the ratio of traveler numbers of Class 1 versus Class 2.

State Level	**Low**	**Medium**	**High**
Demand (km)	30,000	50,000	80,000
VTT (CHF/h)	3	6	10
VWT (CHF/h)	5	10	15
Traveler ratio	2:1	1:1	1:3
Capacity of TNC 1 (km)	24,000		
Capacity of TNC 2 (km)	32,000		
Trip average distance (km)	Class 1	5	
	Class 2	15	

Table 3: Heterogeneous scenario settings.

Tables 4 summarizes the results under the heterogeneous setting. The true state "M-2:1-HM-MM" corresponds to medium-demand, traveler ratio of 2:1, followed by the class-specific VTT/VWT configurations (first letter for Class 1 and second for Class 2). As in the homogeneous setting, both TNCs are able to accurately infer the true market conditions, and TNC2 still secures a larger market share with higher profit. These results imply that, under a large market size, the traveler heterogeneity may not have a significant impact on TNCs' inference on the market conditions.

True state	M-2:1-HM-MM	
	TNC1	TNC2
Price (CHF)	0.1757	0.1998
Profit (CHF)	2201.07	3903.29
Market share (%)	40.29	54.70
Final belief (prob.)	M-2:1-HM-MM (1)	M-2:1-HM-MM (1)

Table 4: Heterogeneous scenario results.

Small market scale

In this experiment, we keep all setting the same as Table 2 but scale down both demand and capacity by a factor of 1000. Different from the previous two scenarios, TNCs no longer always infer the true market conditions. To better illustrate the impact of misbelief on the equilibrium outcomes, we also report the relative changes with respect to the full-information scenario in Table 5.

True state	L-L-M		M-L-M		H-L-M	
	TNC1	TNC2	TNC1	TNC2	TNC1	TNC2
Price (CHF)	0.167(−0.5%)	0.183(+0.4%)	0.311	0.340	0.151(−74.1%)	0.222(−63.3%)
Profits (CHF)	0.371(−3.9%)	0.888(+1.9%)	5.583	8.596	1.986(−82.8%)	5.446(−67.1%)
Market share (%)	42.5	57.4	39.5	53.5	28.0(+4.9%)	37.4(+6.1%)
Final belief (prob.)	L-L-M (0.46) L-M-M (0.27) L-H-M (0.27)	L-L-M (0.65) L-H-M (0.18) L-H-M (0.17)	M-L-M (1)	M-L-M (1)	M-M-L (0.09) M-H-L (0.91)	M-H-H (1)

Table 5: Results under small market scale.

As in Table 5, both TNCs can only correctly estimate the true market state at the medium demand, where most service capacity is used to serve the majority of demand. When demand is low, the surplus capacity enables TNCs to capture the true demand level and travelers' VWT, though not support a reliable estimate of VTT. Nevertheless, the misbelief does make a significant influence. The market shares remain the same, while TNC2 achieves a slightly higher profit (+1.9%) and TNC1 endures a minor decrease in profit (-3.9%). In contrast, when demand is high, neither TNC is capable of inferring the true market state, even a single state variable. Such a misbelief also yields significant deviation from the optimal operational strategies. Consequently, the two TNCs enter a series pricing game and end up with substantial profit loss.

Overall, these findings align well with fundamental statistical principles, that larger samples provide richer information and greater certainty, while small samples tend to cause belief distortions.

Incorrect prior knowledge

We finally test two scenarios where TNCs' assumption on the potential market state is not even correct. The first mimics the homogeneous setting as per Table 1, whereas the true demand is not one of the three possible values. The second corresponds to the heterogeneous setting as per Table 3, though both TNCs assume travelers are instead homogeneous.

True demand (km)	40,000		70,000	
	TNC1	TNC2	TNC1	TNC2
Price (CHF)	0.168(−0.6%)	0.174(−7.0%)	0.582 (+6.5%)	0.606(+6.0%)
Profits (CHF)	1212.5(−6.7%)	2071.5(−13.2%)	10962.5(+1.2%)	15660.0(+0.7%)
Market share (%)	41.2(−1.0%)	58.3 (+0.8%)	25.2 (−1.4%)	34.2(−1.8%)
Final belief (prob.)	M-L-L (1)	L-L-L (1)	H-L-M (1)	H-L-M (1)

Table 6: Results under incorrect demand assumption.

Table 6 reports results under incorrect demand assumptions. When the true demand is 40,000 km, the TNCs form divergent beliefs, i.e., TNC1 infers the market is at medium demand of 50,000 km while TNC2 believes it is at a low demand of 30,000 km. As a result, the profit loss of TNC2 from the full-information scenario is about double the loss of TNC1. When the true demand is 70,000 km, both TNCs overestimate the demand as 80,000 km and thus set higher prices than the full-information scenario, similar to those in the high demand scenario (see Table 2). This optimistic market perception, however, does not harm neither TNCs but instead bring rather a bit higher profit to both of them.

	TNC1	TNC2
Price (CHF)	0.212(+20.3%)	0.283(+41.8%)
Profits (CHF)	3234 (+46.3%)	6619(+70.5%)
Market share (%)	40.9 (+0.6%)	53.4 (−1.2%)
Final belief (prob.)	M-M-H (1)	H-H-M (1)

Table 7: Results under incorrect class assumption.

Table 7 presents the results under the incorrect assumption about traveler heterogeneity. It is interesting to observe that such a misperception of the market actually mitigates competition between TNCs and thus leads to significantly higher profits for both. This finding also aligns well with the conclusion in a recent work that "perfect information leads to perfect competition" (Q. Liu & Chow, 2022).

4 Conclusion

This work proposes a bi-level framework to model an aggregate multimodal transportation system with competing TNCs and PT. At the upper level, TNCs set trip fares to compete for travelers. At the lower level, travelers choose among all travel options following an MNL model. To better reflect reality, the model incorporates incomplete information: TNCs do not know the true market conditions, including demand size, travelers' value of in-vehicle time, and value of waiting time. Instead, they infer and update their beliefs about the market using Bayes' rule based on observed demand and competitors' past strategies, and then set their pricing strategies accordingly. Numerical experiments are conducted under different market settings. Our results confirm that the market scale significantly affects the accuracy of inference. Another interesting result shows that TNCs may gain from ignorance, as it prevents them from engaging in price wars arising under perfect market knowledge.

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