Assessing Public Transport Fares in Denmark: An Optimal Welfare Approach

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Short summary

Public fare policies are vital for creating efficient, equitable, and sustainable transport systems. This study optimizes a welfare function across flat and distance-based pricing strategies, analysing their distributional impacts and effects on equity. Using a comprehensive mode and destination choice model for weekday trips (<50km) in Denmark, we exhibit that a flat fare of 23 DKK (€3.08) would improve welfare and equity without introducing major operational changes. This is driven by consumer surplus gains and increased market shares of active modes for shorter distances and public transport for longer journeys. These mobility changes result in significant health benefits and reduced environmental costs. However, the socio-spatial distribution of welfare effects is uneven, with most gains seen in rural and suburban areas, and among younger and lower-income groups. Further institutional support may also be needed to help public transport recover from revenue losses.
Keywords: public transport fares, welfare efficiency, health effects, external cost of transport, equity

1 Introduction

Many cities face challenges with public transport systems due to the rise in private car usage. Policymakers are considering new fare structures as a potential solution, but creating optimal fare policies is complex and requires a thorough analysis of societal and welfare impacts to ensure a system with high accessibility and acceptable social costs. Such analysis requires a detailed welfare function that accounts for both direct consumer benefits and indirect effects, such as environmental and public health impacts. Additionally, revenue and capacity effects should be considered.

Recent findings from Sweden (Rubensson et al., 2020) challenge the previous consensus that distance-based fares are superior in terms of equity and welfare performance (Cervero, 1981; Nuworsoo et al., 2009). These findings emphasize the need to assess fare structures within a mix of urban and rural areas and in a European context, where travel profiles differ from those in the US. We conducted a welfare economic analysis for short trips (<50km) in Denmark, examining a fare structure that combines flat and distance-based fees. Using a detailed micro-econometric demand model, we derived a welfare function based on these fare components. This function quantifies accessibility through value-of-time estimates (Rich & Vandet, 2019) and considers health impacts from shifts to active transport modes like walking and cycling. It also includes external cost estimates related to emissions, noise, safety, and climate effects (Santos et al., 2010).

We contribute to the current fare discussion by exploring public transport pricing strategies in Denmark and analyse the welfare implications of distance-based and flat fare combinations. Utilising a comprehensive mode-destination model we further assess the impacts on mobility, external costs, and health benefits of an optimal fare. Our findings indicate that the current fare policy is sub-optimal and that a flat-fee policy would enhance overall welfare and equity. The research also showcases the effectiveness of constraint Bayesian optimization (BO) in efficiently identifying optimal fare strategies, similar to its successful application in other transport pricing problems e.g. congestion pricing (Liu et al., 2024; Tay & Osorio, 2022).

2 Methodology

The methodological approach, illustrated in Figure 1, involves using a destination mode demand model for Denmark to calculate welfare under various public transport fare policies. BO was employed to identify the optimal fare policy that maximizes welfare. Subsequently, we analysed the welfare and equity effects of this optimal policy compared to the basecase.

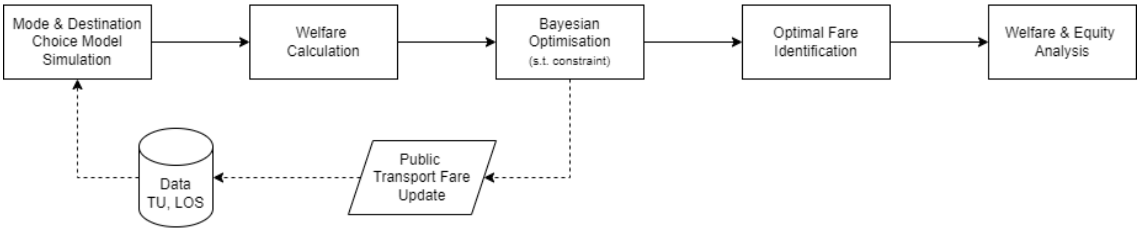

Figure 1: Diagram of the methodological approach

Destination and mode choice model

Considering a large set of daily travel schedules for respondents $n = 1, \ldots, N$, we estimated a demand model to reflect the link between attributes and travel demand probabilities. We considered choice of mode $m = 1, \ldots, M$ and destination $d = 1, \ldots, D$. In total, six different modes were considered: (1) walk, (2) bicycle, (3) e-bike, (4) car, (5) car passenger, and (6) public transport.
The generalised travel time (GTT) for each person n and mode m to each destination zone d was defined as in Eq. (1) similar as in The Danish National Passenger Model (Rich & Hansen, 2016).

$$GTT_n(m,d) = TT_n(m,d) + \left(1 - \delta \frac{q_{(m,p)} - 1}{q_{(m,p)}}\right) \frac{TC_n(m,d)}{VTTS_n} \tag{1}$$

For combining travel time (TT) the travel cost (TC), the latter was transformed by utilising the value-of-travel time savings (VTTS) for each income quartile and adjusted to the 2023 net price index (Rich & Vandet, 2019; Pilegaard et al., 2006). Also note that Eq. (1) applies a cost-sharing formula inspired by Rich & Fox (2024). Hence, we share the cost of trips based on the average car occupation rate $q_{(m,p)}$ for each purpose p. This rate is derived from the Danish National Travel Survey (TU) (Anderson & Hjalmar, 2024), which is described in 3. An equal degree of sharing ($\delta = 1$) between the driver and the passengers is assumed as supported in Rich & Fox (2024).
For the modes: walk, bicycle and e-bike the (TC) is considered zero. For public transport, the cost-sharing part of the function is ignored as costs are already person-based. Utility functions $V_n(m,d)$ are stated in Eq. (2).

$$V_n(m,d) = \beta X_n(m,d) + \log(q_n(d)) + e_n(m,d), \quad \forall n \tag{2}$$

Further, we applied an importance sampling of 20 destinations D based on the zone origin of each trip i. The utility function is adjusted concerning the sampling strategy by including the logarithm of the selection probability $q_n(d|i)$ as described in (Ben-Akiva & Lerman, 1986). Rich & Hansen (2016) demonstrated that sampling provides unbiased estimates of the model parameters.
We estimated a multinomial logit model (MNL) for the combined choice of mode and destination with the probability that individual n chooses mode m and destination d is given by Eq. (3). The software Biogeme was used (Bierlaire, 2023).

$$P_n(m,d) = \frac{\exp(V_n(m,d))}{\sum_{m'=1}^{M}\sum_{d'=1}^{D}\exp(V_n(m',d'))}, \quad \forall n \tag{3}$$

Public transport fares and welfare

Distance and flat fare policies for public transport are expressed in Eq. (4), with distance being measured in kilometres.

$$c_n(m=6,d) = c_{flat} + c_{dist}Distance, \quad \forall n \tag{4}$$

We define $c_{0n} = c_{0n}(m,d)$ as the fare price function that represents the baseline fare policy for public transport. Further, alternative policies $c_{1n} = c_{1n}(m,d|c_{flat}, c_{dist})$ all inherited the structural form from Eq. (4) and is essentially a balance between c_{flat} and c_{dist}.
This gives rise to a person-specific surplus function $Z_n(c_{0n}, c_{1n})$, which measures the net change in the monetary surplus from c_{0n} to c_{1n} according to the rule-of-the-half approximation in Eq. (5).

$$Z_n(c_{0n}, c_{1n}) = \left(\frac{P_n(m,d|c_{1n}) + P_n(m,d|c_{0n})}{2} \right) (c_{0n} - c_{1n}), \quad \forall n \tag{5}$$

The corresponding welfare function representing welfare changes at the individual level is given by Eq. (6).

$$W_n(c_{0n}, c_{1n}) = Z_n(c_{0n}, c_{1n}) + R_n(c_{0n}, c_{1n}) + E_n(c_{0n}, c_{1n}), \quad \forall n \tag{6}$$

$R_n(c_{0n}, c_{1n})$ represents the public transport revenue effect for individual n. $E_n(c_{0n}, c_{1n})$ represents the net external cost effect of the scenario compared to the baseline. This is essentially a scaling of mileage differences multiplied with external cost parameters (Table 1).
Lastly, the calculation of total daily welfare change in Eq. (7) is based on population weights S_n to scale the welfare assessment to a representative Danish population sample.

$$W(c_1(c_{flat}, c_{dist})|c_0) = \sum_{n=1}^{N} S_n W_n(c_{0n}, c_{1n}) \tag{7}$$

Here, we made it explicit that the welfare function W is expressed and optimised over generic values of c_{flat} and c_{dist} relative to the current baseline c_0.

Bayesian optimisation

BO is an approach that by learning the underlying objective function space can provide approximate solutions to problems that are difficult to solve analytically. Employing BO, we could pinpoint regions that optimise total daily welfare under a new fare policy $c_1 = c_1(c_{flat}, c_{dist})$. The BO combines a Gaussian Process (GP) regression (Schulz et al., 2018) and an acquisition function (Frazier, 2018).
The GP regression is stated in Eq. (8), and it is specified by a mean function $\mu(c_1)$ and covariance function $\kappa(c_1, c_1')$. The mean function represents the expected value of the objective function (i.e. the welfare function) at a given fare, and the covariance function (i.e. Matérn kernel) measures the correlation between pairs of fares (c_1, c_1').

$$W(c_1) \sim GP(\mu(c_1), \kappa(c_1, c_1')) \tag{8}$$

The acquisition function ($\alpha(c_1)$) guides the selection of the candidate value $\hat{c_1}$ by balancing exploration and exploitation as in Eq. (9).

$$\hat{c_1} = \underset{c_1}{\operatorname{argmax}}\, \alpha(c_1) \tag{9}$$

Specifically, the constraint expected improvement EI_L, introduced by Gardner et al. (2014) to incorporate constraints in the optimisation process, was used as the acquisition function. This method also models the constraint as a GP regression.

Assumptions

In optimising public transport fares, we assume stable public transport operations, treating the total operational cost as fixed. We accept that the system can handle marginal (i.e. 1%) changes in the average operational cost per traveller, i.e. in the number of travellers, without significant disruptions like overcrowding, inefficiencies, or excessive induced costs. Nevertheless, after determining the optimal fare, we will discuss potential supply effects and limitations.
Our approach is similar to other public transport fare comparison methods, where origin-destination travel demand and revenue were considered fixed, found in the literature (Rubensson et al., 2020; Bureau & Glachant, 2011). However, in contrast to a revenue-neutral with fixed demand fare comparison, which might be overly restrictive given that public transport systems are typically heavily subsidised and not primarily revenue-oriented, our approach can address often-overlooked substitution effects and travel profile changes, along with their associated welfare effects under different fare scenarios. Hence, it allows us to determine possible welfare gains by altering the average travel profile of public transport users, without changing the total number of travellers.

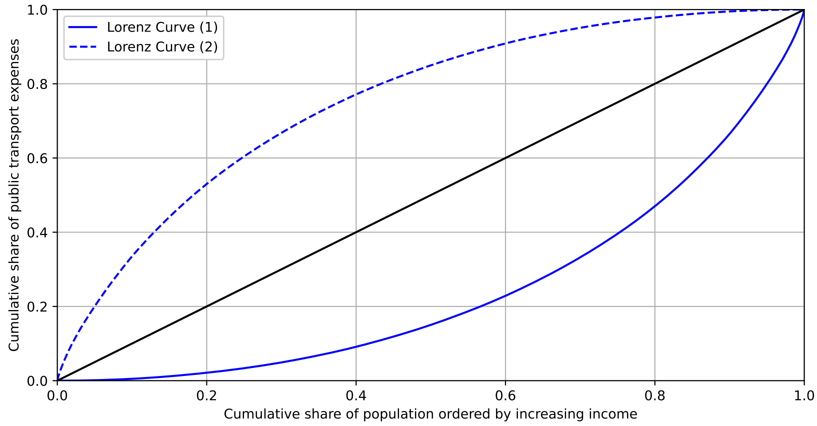

Figure 2: Lorenz curves illustration for calculating the Suits Coefficient. Curve (1) illustrates that low-income groups have a smaller share of expenses in public transport while (2) demonstrates the opposite trend.

Equity index

A commonly used equity index is the Suits coefficient (S) (Suits, 1977). It is calculated based on the area between the Lorenz curve and the horizontal axis (L) and the triangle defined from the diagonal line (K) as in Eq. (10). The horizontal axis represents the cumulative percentage of the population, ordered from lowest to highest income, while the vertical axis shows the cumulative percentage of total paid fares (Figure 2).

$$S = 1 - \frac{L}{K} \tag{10}$$

The index can be either positive or negative, with negative values indicating that the majority of fares are paid by lower-income groups.

3 Results and discussion

Data for Denmark

Table 1: External cost in DKK per kilometre of different transport modes for 2023

	Bicycle & E-bike	Car	Bus	Train (electric)	Train (diesel)
Air pollution	0.000	0.050	0.524	0.022	3.435
Climate change, high CO2 price	0.000	0.127	0.802	0.373	3.164
Noise	0.000	0.089	0.401	0.594	0.594
Accidents	1.998	0.375	0.833	5.161	5.161
Health effects	-8.788	0.000	0.000	0.000	0.000
Total	-6.790	0.641	2.559	6.150	12.353

The TU continuously tracks the travel habits of Danish residents aged 6 and older (Anderson & Hjalmar, 2024). It is based on a representative random sample of one-person, one-day interviews conducted year-round. Specifically, we utilised data from 2015 to 2023, focusing on the primary trip of each journey. Our analysis is limited to weekday transport demand, considering only trips shorter than 50 km. Noteworthy, COVID-19 lockdown periods were also excluded.
A total of 56,277 trips were considered. We categorised trips into three main purposes: 1) "work" includes trips to workplaces, 2) "education" encompasses trips for educational purposes, and 3) "other activities" cover recreational, shopping, and escorting purposes. Business trips exhibit a markedly different travel profile while at the same time representing a limited share of public transport and are therefore disregarded in this research. The most disaggregated regional zonal level containing 3,670 zones was used.

Table 2: Multinomial logit model estimation across different travel purposes for the modes: (1) walk, (2) bicycle, (3) e-bike, (4) car, (5) car passenger, and (6) public transport.

Parameter	Work			Education			Other Activities		
	Value	Std. err.	p-value	Value	Std. err.	p-value	Value	Std. err.	p-value
ASC (1)	2.000	0.298	0.000	-1.380	0.455	0.002	0.682	0.163	0.000
ASC (2)	-2.380	0.535	0.000	-4.730	0.657	0.000	-3.740	0.305	0.000
ASC (3)	-5.660	0.604	0.000	-8.810	0.834	0.000	-7.210	0.356	0.000
ASC (5)	-0.853	0.233	0.000	-1.510	0.633	0.017	-1.180	0.154	0.000
ASC (6)	2.540	0.219	0.000	3.220	0.420	0.000	0.479	0.273	0.079
ln(No. High Edu. Places)	0.052	0.005	0.000	0.199	0.009	0.000	-0.019	0.005	0.000
ln(No. Lower Edu. Places)	0.040	0.004	0.000	0.478	0.012	0.000	0.057	0.003	0.000
ln(No. Workplaces)	0.001	0.000	0.000	0.000	0.000	0.000	0.459	0.008	0.000
ln(No. Summer Houses)	-0.014	0.005	0.006	-0.046	0.010	0.000	0.055	0.004	0.000
ln(Pop. Dens.) (1)	0.054	0.021	0.009	0.304	0.026	0.000	0.224	0.011	0.000
ln(Pop. Dens.) (2)							0.028	0.010	0.006
ln(Pop. Dens.) (3)	-0.067	0.025	0.007						
ln(Pop. Dens.) (5)							0.087	0.009	0.000
ln(Pop. Dens.) (6)	0.069	0.018	0.000	0.049	0.018	0.005	0.316	0.027	0.000
Age: 0 –18 (1)	3.720	0.250	0.000	3.670	0.194	0.000	2.930	0.113	0.000
Age: 0 –18 (2)	3.120	0.241	0.000	4.200	0.232	0.000	3.540	0.112	0.000
Age: 0 –18 (3)	2.010	0.621	0.001	1.240	0.583	0.033	1.820	0.387	0.000
Age: 0 –18 (5)	4.420	0.234	0.000	5.760	0.514	0.000	4.540	0.112	0.000
Age: 0 –18 (6)	3.300	0.280	0.000	4.250	0.239	0.000	4.440	0.149	0.000
Age: 18 –29 (1)	0.493	0.104	0.000				0.213	0.056	0.000
Age: 18 –29 (2)				0.661	0.219	0.003			
Age: 18 –29 (3)	-0.702	0.257	0.006				-0.615	0.269	0.022
Age: 18 –29 (5)	0.744	0.142	0.000	1.620	0.531	0.002	0.593	0.082	0.000
Age: 18 –29 (6)	0.412	0.089	0.000	1.120	0.232	0.000	0.312	0.121	0.010
Age: 54 –74 (1)				-7.550	0.588	0.000	-0.187	0.048	0.000
Age: 54 –74 (2)							0.122	0.052	0.018
Age: 54 –74 (3)	0.325	0.154	0.035				1.020	0.140	0.000
Age: 54 –74 (5)				-5.820	0.555	0.000	0.539	0.057	0.000
Age: 54 –74 (6)							0.312	0.099	0.002
Age: 74 + (3)							0.957	0.209	0.000
Age: 74 + (5)							0.643	0.078	0.000
Age: 74 + (6)				-2.550	1.040	0.014	0.325	0.129	0.012
Bike (2, 3)	5.610	0.503	0.000	4.960	0.508	0.000	4.680	0.260	0.000
1 Car (1)	-3.320	0.176	0.000	-2.920	0.390	0.000	-3.440	0.110	0.000
1 Car (2)	-3.210	0.154	0.000	-2.840	0.380	0.000	-3.470	0.115	0.000
1 Car (3)	-2.980	0.230	0.000	-3.140	0.599	0.000	-3.390	0.177	0.000
1 Car (5)	-2.640	0.223	0.000	-2.030	0.423	0.000	-2.420	0.126	0.000
1 Car (6)	-3.970	0.160	0.000	-2.860	0.378	0.000	-4.550	0.131	0.000
2 + Cars (1)	-4.400	0.198	0.000	-4.150	0.402	0.000	-4.060	0.117	0.000
2 + Cars (2)	-4.580	0.162	0.000	-3.670	0.385	0.000	-4.200	0.122	0.000
2 + Cars (3)	-4.370	0.272	0.000	-3.580	0.729	0.000	-4.250	0.225	0.000
2 + Cars (5)	-3.850	0.239	0.000	-2.630	0.425	0.000	-2.750	0.130	0.000
2 + Cars (6)	-5.780	0.190	0.000	-3.820	0.386	0.000	-5.460	0.159	0.000
Female (1)	0.588	0.091	0.000	0.431	0.158	0.006	0.655	0.042	0.000
Female (2)	0.426	0.057	0.000	0.372	0.146	0.011	0.647	0.047	0.000
Female (3)	1.390	0.153	0.000	1.170	0.440	0.008	1.140	0.125	0.000
Female (5)	0.783	0.110	0.000	0.334	0.153	0.029	1.320	0.048	0.000
Female (6)	0.481	0.077	0.000	0.366	0.151	0.015	0.628	0.077	0.000
GTT (1)	-0.064	0.005	0.000	-0.153	0.007	0.000	-0.147	0.004	0.000
GTT (2)	-0.109	0.002	0.000	-0.202	0.005	0.000	-0.197	0.004	0.000
GTT (3)	-0.085	0.005	0.000	-0.135	0.019	0.000	-0.162	0.010	0.000
GTT (4,5,6)	-0.051	0.002	0.000	-0.016	0.003	0.000	-0.066	0.003	0.000
ln(GTT) (4,5,6)	-0.455	0.054	0.000	-2.300	0.078	0.000	-1.360	0.047	0.000
LL			-38,691			-12,077			-64,800
No. Observations			12,355			5,164			24,687

We extended the TU data with zonal and Level-of-Service (LOS) information for 2023 (sources: Danish Road Directorate, Danish Statistical Bureau, Central Register of Buildings and Dwellings (BBR), Danish Tax Agency), detailing: travel time and cost between zones, population figures, workplaces, and educational institutions per zone, employee numbers per zone and sector, number of students, and number of vacation houses.

External costs prices are presented in Table 1 and are consistent with Pilegaard et al. (2006). For walking, in the absence of recommendations in Denmark, the values were considered the same as those for bicycles and e-bicycles, which can be seen as a conservative assumption (Gössling et al., 2019). Lastly, the estimated models and their associated variables are presented in Table 2. Coefficients not significant in the 95% level were omitted from the models.

Basecase policy

We approximated the basecase policy of our analysis by estimating a linear regression model similar to Eq. (4) over the observed public transport fares. The baseline combines a 13.1 DKK flat fee with a 0.92 DKK/km distance charge and closely replicates the current zone-based pricing scheme ($R^2 = 0.74$).
The mode shares for the basecase are as follows: walking at 15.73%, cycling at 18.73%, e-biking at 1.37%, driving at 46.39%, car passengers at 10.75%, and public transport at 7.12%. The average trip lengths for each mode are 2.49 km for walking, 6.22 km for bicycling, 9.18 km for e-biking, 28.2 km for driving, 23.25 km for car passengers, and 23.16 km for public transport.

Optimal fare

The Bayesian optimisation ran for 50 iterations and identified the optimal fare as 23 DKK flat fare. In Figure 3 the coloured area illustrates the combination of distance and flat fare parameters that cause a marginal average operational cost per traveller (i.e. maximum 1% change in the number of total travellers) based on the estimated welfare and constraint GP regressions. We observe, that as the travel profiles change along the line, welfare benefits are generated when prices move towards heavier flat fare policies. Compared to the basecase fare adopting a 23 DKK flat fare generates a welfare gain of 0.34 DKK/person. That roughly results in a total of 1 mil. DKK per day. It is worth mentioning that the flat fee would improve the consumer surplus and external cost by 0.38 and 0.095 DKK/person, respectively, however, it will introduce public transport revenue losses of 0.13 DKK/person.

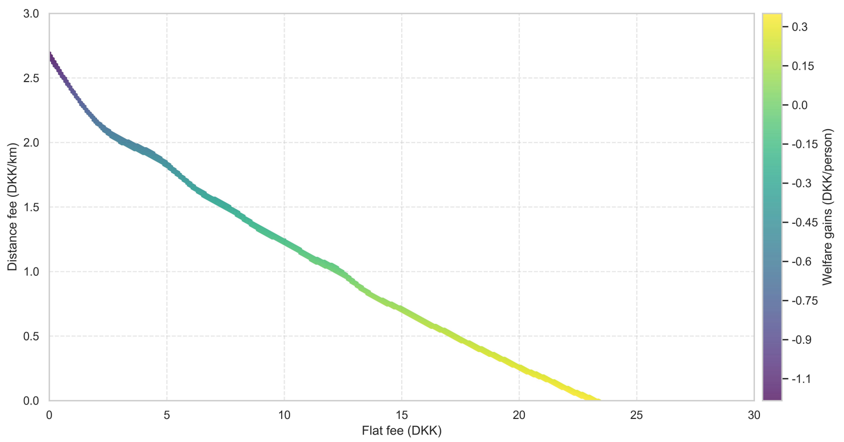

Figure 3: Bayesian optimisation (BO) of public transport fares. The welfare gains are calculated compared to the basecase fare (13.1 DKK & 0.92 DKK/km). The coloured area represents fares that account for a maximum of 1% change in average operational cost per traveller compared with the basecase.

Demand, welfare, and equity effects

Examining the distributional changes in mileage for different modes the 23 DKK flat fare leads to a 0.83%, 0.06%, -0.14%, -0.53%, -1.99%, and 28.33% change in total mileage for walking, bike, e-bike, car, car passenger, and public transport, respectively. In Figure 4, we see an increase in both walking and bicycling for trips below 8 kilometres, significantly contributing to health benefit gains. On the other hand, public transport become more competitive for longer distances, substituting car trips above 10 km. This also results in external cost gains from reduced emissions, noise, and accidents. Lastly, the e-bike mode overall presents a slight decrease in the total mileage, showcasing also its different distance profile compared to regular bicycles.

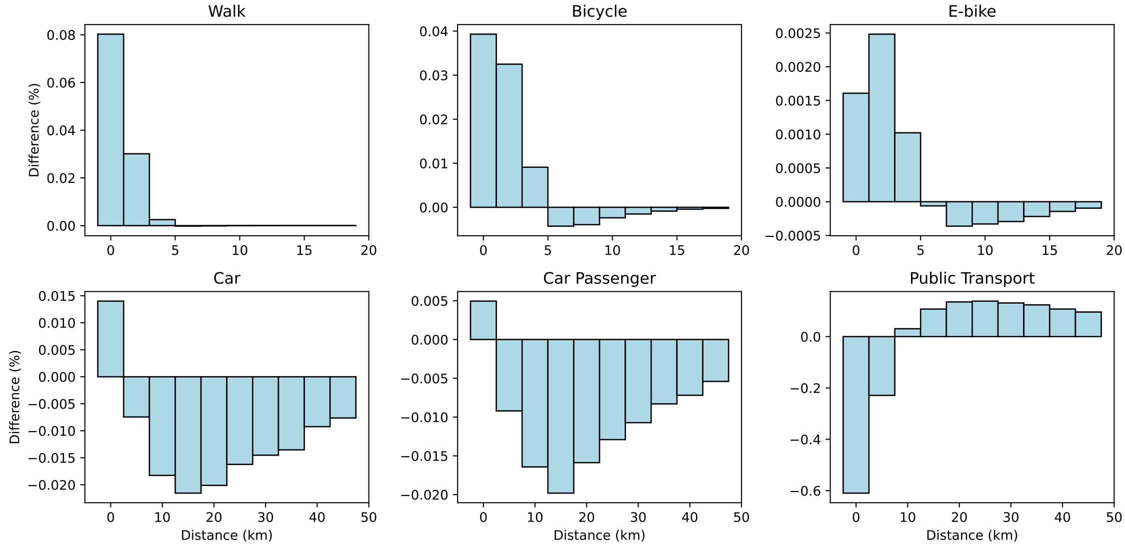

Figure 4: Mileage distributional effects (% change from the basecase) across modes and travel distances of implementing the 23 DKK flat fare scheme for public transport.

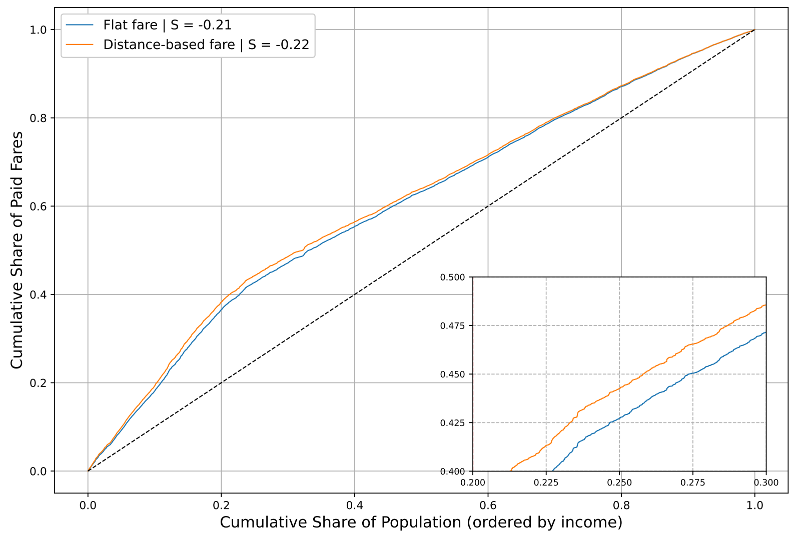

Figure 5: Lorenz curves for current zonal-based fares, distance-based fares (13.1 DKK & 0.92 DKK/km) and flat fares (23 DKK).

Furthermore, based on our examination of the welfare distributional effects, we found that the greatest benefits are experienced by suburban and rural areas, while dense urban zones face a negative welfare impact. Additionally, our analysis across socioeconomic groups revealed that younger people, those travelling for educational purposes, and lower-income segments exhibit the largest welfare gains (around 1 DKK/person).
Lastly, regarding equity, we calculated the Suits coefficient from the Lorenz curves, shown in Figure 5, for the basecase and the welfare-optimal flat fare. We observe that public transport fares are predominantly paid by lower-income population segments, with a coefficient of around -0.2. Among the different fare structures, the flat fare demonstrates an improvement in equity, with a 5% increase from the basecase fare regime.

Discussion and limitations

A transition to a public transport flat fare in Denmark, similar to the finding of Rubensson et al. (2020), has a positive effect on equity, with low-income groups and younger people benefiting the most. Such a policy could be advantageous, as it supports the more vulnerable segments of society by making public transport more accessible and affordable. Also, aligned with Nishiuchi et

al. (2024), there is a positive welfare effect in suburban areas by enabling people to travel longer distances. Our results correspond to the research for European cities and rural areas while being less aligned with findings from American cities (Nuworsoo et al., 2009; Farber et al., 2014).
It is important to acknowledge that the proposed flat fare policy negatively impacts welfare in dense urban areas by reducing public transport usage. However, shifting towards more active modes of transport could be beneficial, especially since urban centres are often the main public transport bottlenecks. In turn, improved comfort and fare simplicity for public transport might attract new users to public transport (e.g. car users). These kinds of effects were not covered in this study.
Additionally, the increase in public transport usage in suburban and rural areas should be manageable with the current system, as the main supply surplus is commonly found in decentralised areas. Although our study is limited in accounting for these potential supply effects, we believe that major supply constraints would not arise from this shift. However, a supply model could reveal more complex welfare effects and be a valuable extension for future research.

4 Conclusions

We demonstrate that welfare in Denmark can be improved with a new public transport fare, using a constrained BO approach over a flexible fare structure, compared to the basecase, for weekday trips (<50 km). Further, the identified flat fare of 23 DKK ($\sim$ 3.08€) offers several benefits: it makes public transport more competitive with car travel over long distances, promotes active mobility for short distances, and improves accessibility and affordability for younger and lower-income populations. Moreover, Bayesian optimization is a particularly valuable tool for designing public transport fare policies, even under demand constraints and could become even more in large-scale transport models with high computational costs. Overall, we suggest that a well-designed fare structure can balance environmental benefits, health gains, and social equity, making public transport more attractive and sustainable.

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