Mixed Integer Formulation with Linear Constraints for Integrated Service Operations and Traveler Choices in Multimodal Mobility Systems

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SHORT SUMMARY

Multimodal mobility systems provide seamless travel by integrating different types of transportation modes. Most existing studies model service operations and travelers' choices independently or limited in multimodal travel options. We propose a choice-based optimization model for optimal operations of multimodal mobility systems with embedded travelers' choices using a multinomial logit (MNL) model. We derive a mixed-integer linear formulation for the problem by linearizing transformed MNL constraints with bounded errors. The preliminary experimental test for a small mobility on demand and public transport network shows the model provides a good solution quality.

Keywords: Integrated service operations and user choices, Linearization of discrete choice constraints, Multimodal mobility systems.

1 INTRODUCTION

Multimodal mobility systems integrate different modes of transportation, such as walking, cycling, driving, public transportation, and ride-sharing services, into a seamless and efficient network. Recent advances in autonomous vehicles have the potential to increase coordination among traffic modes, especially for Mobility-on-Demand (MoD) services (e.g., taxis, Lyft, Uber, and DiDi) which provide point-to-point services and can connect travelers to public transportation.

In the multimodal mobility area, from the supply side, service providers decide on operations like vehicle routing. For the demand side, travelers choose the modes (e.g., Subway, ride-sharing services, or a combination of them) and path according to the features of available options like travel time and price. However, most studies model service providers' operations and travelers' choices independently or limited in multimodal travel options. For example, Wollenstein-Betech et al. (2022) proposed an integrated Autonomous Mobility-on-Demand (AMoD) system with public transportation. They optimize the routing and rebalancing of the AMoD fleet from the system-optimum perspective, while the travelers' choices of modes are exogenous to the operation optimization model. Liu et al. (2019) developed a multimodal transportation system integrating a choice model in which travelers can choose either public transport or MoD services but not multimodal travel options. Pi et al. (2019) integrated a choice model in a multimodal dynamic traffic assignment model by repeatedly updating travelers' pre-defined multimodal mode choices and assignment results until convergence.

Conceptually, an effective operation modeling in a multimodal mobility system should jointly consider service operations (supply) and travelers' choice preferences (demand). Mathematically, this

can be modeled as a choice-based optimization problem. Choice-based optimization models are mathematical models that optimize decision-making based on choices made by individuals. These models are commonly used in marketing, economics, and other fields where individuals make choices based on various factors such as price, quality, and convenience. Many studies found that the choice-based optimization model is highly relevant in the real world since it can provide more accurate predictions of decision-making behavior Roemer et al. (2023).

The paper proposes a choice-based optimization model for cooperative travels in the multimodal mobility system, which aims to minimize the total system travel time by deciding part of service operations while satisfying travelers' choice preferences characterized by the multinomial logit model (MNL). Due to the non-linearity and non-convexity of the MNL model embedded in the problem, limited research has thus far been devoted to solving it. Pacheco Paneque et al. (2021) proposed a mixed-integer linear formulation based on simulation for the related discrete choice models. Later, Pacheco Paneque et al. (2022) adopted scenario decomposition and scenario grouping based on their aforementioned paper into a novel Lagrangian decomposition method to solve a choice-based optimization problem.

Different from the simulation and the Lagrangian decomposition-based methods, we propose and explore a new mixed-integer formulation for the choice-based optimization model for the studied problem. The main contributions of this paper are two-fold:

- Propose a choice-based optimization model for cooperative travels in multimodal mobility systems. It optimizes system travel times by deciding part of service operations while satisfying travelers' choice preferences.

- Propose a novel mixed-integer formulation to effectively solve the choice-based optimization problem by linearizing transformed MNL constraints with bounded errors.

Note that we validate our model on a simple network with MoD and public transport services and compared it with sampling and simulation-based approach in linearizing MNL constraints. More experimental tests on the real-world network will be conducted and also compared with state-of-art models and general nonlinear optimization solvers, such as numerical optimization and meta-heuristics.

2 METHODOLOGY

Model description

We define a multi-modal transportation network G containing multiple layers. One layer represents MoD services, and each other layer can represent a specific mode, such as subway, buses, shared bikes, or walking. For simplicity, we only discuss one MoD layer and one public transportation layer (e.g., subway), as shown in Fig 1. However, our formulation can be easily expanded to one MoD layer and multiple other layers.
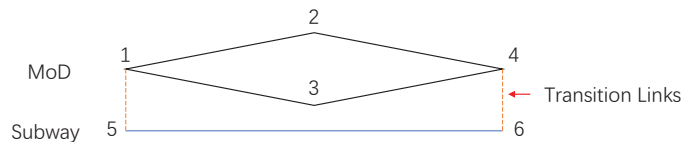

Figure 1: A simple example of a multi-modal transportation system

Denote $G = \{G_m \cup G_p\}$ that MoD layer $G_m = (V_m, E_m)$ has vertices V_m and edges E_m. The same for the public transportation layer $G_p = (V_p, E_p)$. There are transition links T connecting two different layers to represent possible transfers of modes. Denote $G = (V, E)$ that $V = \{V_m \cup V_p\}$ is the set of all points in the network and $E = \{E_m \cup E_p \cup T\}$ is the set of all edges in the network.

To model demand over different OD pairs, denote by $w = (w_s, w_t)$ an OD pair starting from vertex w_s to vertex w_t and $d_w \geq 0$ as the travel demand rate in this OD pair. Define W as the set of OD pairs. We denote by x_e^w the travel flow by OD pair w on edge e, $x_e = \sum_{w \in W} x_e^w$ the total flow on edge e, and x_e^b the rebalancing flow of MoD services. $\boldsymbol{X} = \{x_e^w | e \in E, w \in W\}$ and

$\boldsymbol{X^b} = \{x_e^b | e \in E\}$ are non-negative continuous decision variables.

For OD pair w, denote R^w as the set of possible routes and θ^{wr} as the percentage of travelers taking route r. To model relationships between edges and routes, define δ_{er}^w as equal to 1 if edge e belongs to route r of OD pair w, 0 otherwise. Along with each edge $e \in E$, we also have associated non-negative travel time $t_e \geq 0$ and non-negative cost $p_e \geq 0$. We can set travel time and prices for edges to express different travel time patterns and price policies. The travel time t^r and cost p^r can then be expressed simply by the equations

$$t^r = \sum_{e \in E} \delta_{er}^w t_e \quad \forall w \in W, \ \forall r \in R^w, \tag{1}$$

$$p^r = \sum_{e \in E} \delta_{er}^w p_e \quad \forall w \in W, \ \forall r \in R^w. \tag{2}$$

We assume the utility function of route r for the OD pair w as follows:

$$\mu^{wr} = -\beta_1 t^r - \beta_2 p^r, \tag{3}$$

where β_1 and β_2 are marginal costs for time and price respectively.

To model our problem, denoted $\boldsymbol{P_0}$, we define $E^+(i) \in E$ as the set of edges starting from vertex i, and $E^-(i) \in E$ as the set of edges ending with vertex i. Then, $\boldsymbol{P_0}$ can be expressed as follows:

$$\min_{\boldsymbol{X}, \boldsymbol{X^b}} \sum_{e \in E} t_e (x_e) x_e \tag{4}$$

$$\text{s.t.} \sum_{e \in E^-(i)} x_e^w + 1_{i=w_s} d_w = \sum_{e \in E^+(i)} x_e^w + 1_{i=w_t} d_w \quad \forall w \in W, \ i \in V, \tag{5}$$

$$\sum_{e \in E_m^-(i)} \left(x_e^b + x_e \right) = \sum_{e \in E_m^+(i)} \left(x_e^b + x_e \right) \quad \forall i \in V_m, \tag{6}$$

$$x_e^w = d_w \sum_{r \in R^w} \delta_{er}^w \theta^{wr} \quad \forall w \in W, \ \forall e \in E, \tag{7}$$

$$\theta^{wr} = \frac{\exp \left(\mu^{wr} \right)}{\sum_{r' \in R_w} \exp \left(\mu^{wr'} \right)} \quad \forall r \in R^w, \ \forall w \in W, \tag{8}$$

$$x_e^w \geq 0 \quad \forall w \in W, \ \forall e \in E, \tag{9}$$

$$x_e^b \geq 0 \quad \forall e \in E. \tag{10}$$

Objective (4) minimizes total travel time in the multi-modal system. Constraint (5) complies with flow conservation and demand. Constraint (6) regulates the rebalancing flow of MoD service. Constraint (7) describes the relationship between flow and route. Constraint (8) ensures the percentages of routes' choices fulfill an MNL model. Constraints (9) and (10) define non-negative ranges for the decision variables.

Linearizing transformed MNL constraints

The main computational challenge of the model is due to the nonlinear parts, especially the non-linear constraint (8). Based on three reasonable assumptions, we are able to separately deal with distinct cases for constraint (8) to avoid some computational challenges.

For OD pair w, we select an arbitrary route r_0 as the base route. Then, constraint (8) can be represented by the constraints

$$\frac{\theta^{wr}}{\theta^{wr_0}} = \frac{\exp \left(\mu^{wr} \right)}{\exp \left(\mu^{wr_0} \right)}, \forall r \in R^w / r_0, \forall w \in W, \tag{11}$$

$$\sum_{r \in R^w} \theta_r = 1, \forall w \in W. \tag{12}$$

.
Take the natural logarithm on both sides of constraint (11):

$$\ln \theta^{wr} - \ln \theta^{wr_0} = \mu^{wr} - \mu^{wr_0}, \forall r \in R^w / r_0, \forall w \in W \tag{13}$$

Then, We give an equivalent formulation $\boldsymbol{P_0'}$ for our original problem: (4), s.t.{(5), (6), (7), (9), (10), (12), (13)}

In $\boldsymbol{P_0'}$, constraint (13) is still non-linear and could be handled by a piece-wise linear approximation. However, for $\varphi = ln\theta$, it is difficult to obtain accurate piece-wise linearization when θ is close to 0 since $\varphi' = 1/\theta \to +\infty$, when $\theta \to 0$. Therefore, we apply slight transformations of the original problem to avoid that θ takes a value close to 0 when doing piece-wise linearization.

Constraint (8) describes the MNL model. We revise the model by adding three assumptions for an OD pair w:

1. θ^{wr} can only take values of $0 \cup [\epsilon, 1]$ where ϵ is a small threshold, such as 0.1%, that fulfills the accuracy requirements of the application.

2. if $\theta^{wr_1} \geq \epsilon$ and $\theta^{wr_2} \geq \epsilon, r_1, r_2 \in R^w$, then:

$$\frac{\theta^{wr_1}}{\theta^{wr_2}} = \frac{\exp\left(\mu^{wr_1}\right)}{\exp\left(\mu^{wr_2}\right)}. \tag{14}$$

3. if $\theta^{wr} = 0$, then $\forall \theta^{wr'} \geq \epsilon, \ r' \in R^w, \ r' \neq r :$

$$\theta^{wr'} \frac{\exp\left(\mu^{wr}\right)}{\exp\left(\mu^{wr'}\right)} < \epsilon. \tag{15}$$

We argue that these assumptions are reasonable. In practice, the original MNL model assigns extremely small probabilities to options/routes no matter how inferior they are according to constraint (8). It is common to round the probabilities for these unattractive options to 0 as long as they are lower than a threshold similar to assumption 1. Assumption 2 ensures that all non-zero probabilities fulfill the MNL relationship. Assumption 3 ensures that options with 0 probability are unattractive options. Thus, the assumptions allow us to exclude parts of the search space that are not interesting for the application but may create numerical challenges.

We now calculate the error boundary when these three assumptions are used to replace constraint (8). For simplicity, we restrict the discussion to one OD pair and assume there are n options/routes. Denote by $\hat{\theta}_i$ the probability of route i computed based on the assumptions and by θ_i the probability computed by the original constraint. μ_i is the utility function of route i. We define sets N_0 and N_1 that route $i \in N_0$ if $\hat{\theta}_i = 0$, $i \in N_1$ if $\hat{\theta}_i \geq \epsilon$. N is the union of N_0 and N_1.

For route $i \in N_0$:

$$\begin{aligned}
\Delta_i &= |\theta_i - \hat{\theta}_i| = \theta_i \\
&= \frac{\exp\left(\mu_i\right)}{\sum_{j \in N} \exp\left(\mu_j\right)} \leq \frac{\exp\left(\mu_i\right)}{\sum_{j \in N_1} \exp\left(\mu_j\right)} \\
&\text{Since: } \exp\left(\mu_j\right) > \frac{\hat{\theta}_j \exp\left(\mu_i\right)}{\epsilon}, \forall j \in N_1 \\
&< \frac{\epsilon}{\sum_{j \in N_1} \hat{\theta}_i} = \epsilon
\end{aligned} \tag{16}$$

For route $i \in N_1$:

$$\Delta_i = |\theta_i - \hat{\theta}_i|$$

$$= \frac{\exp(\mu_i)}{\sum_{j \in N_1} \exp(\mu_j)} - \frac{\exp(\mu_i)}{\sum_{j \in N} \exp(\mu_j)}$$

$$= \frac{\exp(\mu_i) \sum_{j \in N_0} \exp(\mu_j)}{\sum_{j \in N_1} \exp(\mu_j) \sum_{j \in N} \exp(\mu_j)}$$

$$= \hat{\theta}_i \frac{\sum_{j \in N_0} \exp(\mu_j)}{\sum_{j \in N} \exp(\mu_j)} \leq \frac{\sum_{j \in N_0} \exp(\mu_j)}{\sum_{j \in N} \exp(\mu_j)}$$

$$= \frac{1}{1 + \frac{\sum_{j \in N_1} \exp(\mu_j)}{\sum_{j \in N_0} \exp(\mu_j)}} \leq \frac{1}{1 + \frac{\sum_{j \in N_1} \exp(\mu_j)}{|N_0| \max_{j \in N_0} \exp(\mu_j)}} \quad (17)$$

$$\text{Since: } \exp(\mu_j) > \frac{\hat{\theta}_j \max_{k \in N_0} \exp(\mu_k)}{\epsilon}, \forall j \in N_1$$

$$< \frac{|N_0|\epsilon}{|N_0|\epsilon + \sum_{j \in N_1} \hat{\theta}_j} = \frac{|N_0|\epsilon}{|N_0|\epsilon + 1}$$

$$\leq \frac{|N|\epsilon}{|N|\epsilon + 1}$$

Therefore, with a threshold of an acceptable error bound $\max_i |\Delta_i|$, we can define our new problem $\boldsymbol{P_1}$ with the replacement of constraint (8) by the three assumptions.

To encode the logic implied by the assumptions into our optimization problem, we introduce binary variables b^{wr}. These binary variables work as indicators with $b^{wr} = 1$ if $\theta^{wr} \geq \epsilon$, and 0 otherwise. We assume the absolute value of the utility function (3) has an upper bound $|U|_{max}$. Then, define continuous variables $\varphi^{wr} \in [\ln \epsilon - |U|_{max}, 0], w \in W, r \in R^w$, $\tilde{\varphi}^{wr} \in [\ln \epsilon, 0], w \in W, r \in R^w$, and $\tilde{\theta}^{wr} \in [\epsilon, 1], w \in W, r \in R^w$ for auxiliary. Define a small positive value τ to deal with the strict inequality in assumption 3 and avoid numerical issues in (20).

$\boldsymbol{P_1}$:

$$\min_{\boldsymbol{X}, \boldsymbol{X^b}} \sum_{e \in E} t_e(x_e) x_e$$

$$s.t. \ (5), (6), (7), (9), (10), (12),$$

$$\theta^{wr} = 0 \cup [\epsilon, 1] \quad \forall w \in W, \ r \in R^w, \quad (18)$$

$$b^{wr} \geq \theta/2 \quad \forall w \in W, \ r \in R^w, \quad (19)$$

$$b^{wr} \leq \theta/\epsilon + \tau \quad \forall w \in W, \ r \in R^w, \quad (20)$$

$$\varphi^{wr} - \varphi^{wr_0} = \mu^{wr} - \mu^{wr_0} \quad \forall w \in W, \ \forall r \in R^w/r_0, \quad (21)$$

$$\tilde{\varphi}^{wr} = \ln \tilde{\theta}^{wr} \quad \forall w \in W, \ \forall r \in R^w, \quad (22)$$

$$b^{wr} = 1 \rightarrow \varphi^{wr} = \tilde{\varphi}^{wr} \quad \forall w \in W, \ \forall r \in R^w, \quad (23)$$

$$b^{wr} = 1 \rightarrow \theta^{wr} = \tilde{\theta}^{wr} \quad \forall w \in W, \ \forall r \in R^w, \quad (24)$$

$$b^{wr_1} = 0 \text{ and } b^{wr_2} = 1 \rightarrow$$

$$\mu^{wr_1} - \mu^{wr_2} \leq \ln \epsilon - \varphi^{wr_2} - \tau \quad \forall w \in W, \ \forall r_1, r_2 \in R^w, \ r_1 \neq r_2, \quad (25)$$

$$b^{wr} \in \{0, 1\} \quad \forall w \in W, \ \forall r \in R^w, \quad (26)$$

$$\ln \epsilon - |U|_{max} \leq \varphi^{wr} \leq 0 \quad \forall w \in W, \ \forall r \in R^w, \quad (27)$$

$$\ln \epsilon \leq \tilde{\varphi}^{wr} \leq 0 \quad \forall w \in W, \ \forall r \in R^w. \quad (28)$$

Constraints (18), (19), and (20) restrict the ranges of θ^{wr}. They indicate that θ^{wr} is non-zero by $b^{wr} = 1$ and zero by $b^{wr} = 0$. Constraints (21), (22), (23), and (24) ensure assumption 2 holds when probabilities of two routes are greater than ϵ and make piece-wise linearization of natural log function starts from $\ln \epsilon$. Constraints (25) ensure assumption 3 holds. Constraints (26), (27), and (28) define ranges of variables.

In a sophisticated mixed-integer linear programming solver, such as Gurobi, it is possible to include constraint (18) by semi-continuous variables, and constraint (22) through so-called general constraints. The formulation $\boldsymbol{P_1}$, thus, allows us to utilize powerful mixed-integer linear programming software.

3 RESULTS AND DISCUSSION

In this section, we present experiments based on an artificial network to show the accuracy of the results. As shown in Fig (2), the case has two layers of MoD services and Subway connected by some transition links. In the MoD layer, points represent districts in a virtual city while edges are abstracted roads. In Subway layer, there are two lines. Each station is connected with a point in the MoD layer by a transition link.
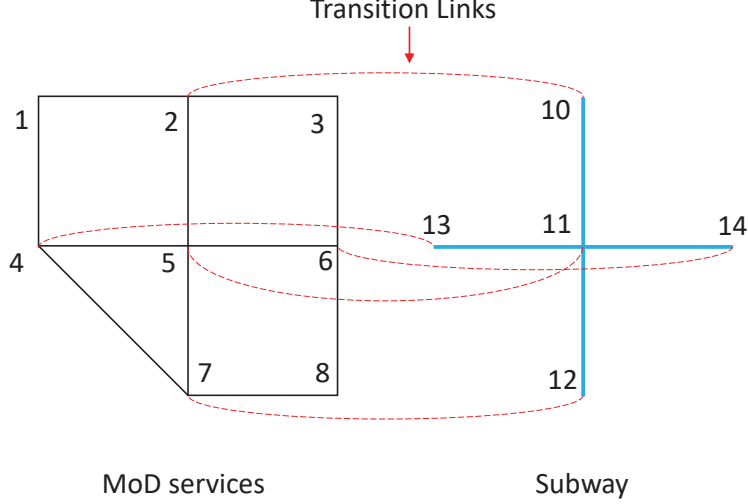

Figure 2: The case illustration

For fare, we set a distance-based fare for MoD services that $p_e = 10\$, \forall e \in E_m$ and an entrance-based fare for the subway that $p_e = 4\$, \forall e = (i,j), i \in V_m, j \in V_p$. As for the travel time, we assume a constant travel time of 5 min for transition links and 15 min for all edges in the public transportation layer. We use Bureau of Public Roads function (29) for edges in the MoD layer, given by

$$t_e\left(x_e\right) = t_e^0 \left(1 + \alpha\left(x_e/m_e\right)^\beta\right), \tag{29}$$

and we use the typical values $\alpha = 0.15$ and $\beta = 4$. $t_0 = 10\sqrt{2}$ min is set in function (29) for edge (4, 7) and $t_0 = 10$ for the rest of edges. We also assume all edges have the same capacity $m = 20k$ pcu/h. We set a fixed time of 15 min for edges in Subway layer and 5 min for transition links. To solve the model, we apply piece-wise linearization to (29). Gurobi can handle the quadratic terms in the objective (4) by "translating them into bilinear form and applying spatial branching" according to its website (https://www.gurobi.com/documentation/10.0/refman/nonconvex.html).

The parameters in the utility function (3) are normally estimated by real data. However, we directly define $\beta_1 = 1/min$ and $\beta_2 = 1/\$$ for two reasons: (1) Our contribution is in the algorithm. Such settings are enough for illustration and basic exploration. (2) For utility function's parameter estimation, most literature on the multi-modal transportation system normally do modal split first to decide demands for each mode. However, our problem jointly considers mode and route choices. It is difficult to find a perfectly suitable estimation in current research.

In the given case settings, we solve $\boldsymbol{P_1}$ with a threshold ϵ of 0.01 by an i9-12900H CPU and Gurobi 10.0.0 in 22.42s. Table 1 displays the solution results of our model. There are 4 OD pairs and corresponding flows as shown by $w : d_w$. $|R^w|$ represents the number of available routes of OD pair w. $r \in R^w$ shows the detailed information of active routes which have non-zero choice probabilities and how many inactive routes. $\hat{\theta}^{wr}$ is the choice probability obtained by $\boldsymbol{P_1}$ and θ^{wr} is the one computed by the original MNL model based on the utility values in the solution. Δ^{wr} is the difference between two computations of choice probabilities to measure the solution quality.

Table 1 shows the solution results of the proposed formulation. The differences Δ^{wr} between the linearized MNL and the original MNL models are quite small, which illustrates that the proposed

Table 1: Results of the proposed formulation

OD: flow	Routes	Route	P_1	MNL	Error
$w : d_w$	$|R^w|$	$r \in R^w$	$\hat{\theta}^{wr}$	θ^{wr}	Δ^{wr}
(1, 8): 15k	22	[1, 4, 7, 8]	100.00%	100.00%	0.00%
		21 routes remain...	0.00%	$\leq 0.00\%$	$\leq 0.00\%$
(8, 1): 30k	22	[8, 6, 3, 2, 1]	4.05%	4.02%	0.03%
		[8, 6, 5, 2, 1]	6.67%	6.65%	0.03%
		[8, 7, 4, 1]	89.27%	89.10%	0.18%
		19 routes remain...	0.00%	$\leq 0.21\%$	$\leq 0.21\%$
(3, 7): 23k	21	[3, 6, 5, 7]	43.71%	43.25%	0.46%
		[3, 6, 14, 11, 12, 7]	7.67%	7.58%	0.09%
		[3, 2, 5, 7]	41.68%	41.24%	0.44%
		[3, 2, 10, 11, 12, 7]	6.94%	6.86%	0.09%
		17 routes remain...	0.00%	$\leq 0.99\%$	$\leq 0.99\%$
(7, 3): 25k	21	[7, 8, 6, 3]	19.15%	19.14%	0.01%
		[7, 5, 6, 3]	29.83%	29.82%	0.01%
		[7, 5, 2, 3]	42.18%	42.17%	0.00%
		[7, 12, 11, 14, 6, 3]	3.25%	3.22%	0.03%
		[7, 12, 11, 10, 2, 3]	5.60%	5.59%	0.01%
		16 routes remain...	0.00%	$\leq 0.03\%$	$\leq 0.03\%$

method gives a good approximation.

We also tried the sampling and simulation method inspired by Pacheco Paneque et al. (2021) to linearize the choice constraint (Table 2). We set 100 draws for each OD pair and solved the same case in 3039.57s. Here, $\hat{\theta}^{wr}$ is the probability obtained by the simulation-based method and θ^{wr} is the one calculated by the MNL model based on the utility values in the solution. We still use the differences between the two probabilities to measure the solution quality.

Table 2: Results of the simulation-based formulation

OD: flow	Routes	Route	Simulation	MNL	Error
$w : d_w$	$|R^w|$	$r \in R^w$	$\hat{\theta}^{wr}$	θ^{wr}	Δ^{wr}
(1, 8): 15k	22	[1, 4, 7, 8]	100.00%	100.00%	0.00%
(8, 1): 30k	22	[8, 6, 3, 2, 1]	4.00%	5.61%	1.61%
		[8, 6, 5, 2, 1]	6.00%	8.98%	2.98%
		[8, 7, 4, 1]	90.00%	85.08%	4.92%
(3, 7): 23k	21	[3, 6, 5, 7]	46.00%	37.67%	8.33%
		[3, 6, 14, 11, 12, 7]	7.00%	7.73%	0.73%
		[3, 2, 5, 7]	40.00%	45.64%	5.64%
		[3, 2, 10, 11, 12, 7]	7.00%	7.96%	0.96%
(7, 3): 25k	21	[7, 8, 6, 3]	18.00%	19.30%	1.30%
		[7, 5, 6, 3]	31.00%	27.02%	3.98%
		[7, 5, 2, 3]	42.00%	44.93%	2.93%
		[7, 12, 11, 14, 6, 3]	4.00%	3.03%	0.97%
		[7, 12, 11, 10, 2, 3]	5.00%	5.66%	0.66%

As shown in Table 2, the simulation-based formulation produces the same routes of non-zero probabilities as the ones by our proposed formulation. However, the error of the simulation-based formulation can be up to 8.33% which is much greater than the error shown in Table 1. The comparison suggests our proposed formulation has good solution quality.

4 Conclusions

This paper proposes a choice-based optimization model for integrated service operations and traveler choices in multimodal mobility systems. We derive a mixed-integer formulation by linearizing the MNL-based discrete choice constraints with bounded errors. Preliminary experiments show that the proposed formulation provides a good solution quality. Future work will derive the computation complexity and test the methodology on large-size problems, as well as compare it with state-of-art solution methods.

Acknowledgements

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