Improved precision in a heuristic for particle-based and stochastic dynamic traffic assignment

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1 Short summary

This work continues Flötteröd (2022) and refines an operational and efficient heuristic for the stochastic and particle-based dynamic traffic assignment problem. Experimental results indicate that the method performs advantageous when compared to alternative techniques that are applicable in the considered "agent-based" setting (variations of the method of successive averages (MSA) and the sorting technique of Sbayti et al. (2007)). Conveniently, the method requires no tuning parameters, not even an MSA-like step size rule.

2 Introduction

2.1 Context

This work builds on and extends Flötteröd (2022), where a mathematical formulation for the stochastic and particle-based dynamic traffic assignment problem and a corresponding solution heuristic were presented. The stochastic and particle-based setting is characteristic for modern "agent-based" transport simulators that trade mathematical tractability against detailed descriptive capabilities (Axhausen et al., 2016; Barcelo, 2010).

Informally, the agent-based traffic assignment (ATA) problem considered here is to assign to every individual in a synthetic traveler population a travel plan (trip sequence annotated with routes, modes, departure times) such that the behavioral model attached to that individual cannot improve upon that plan. This reminds of a Nash equilibrium (NE), only that the exact computation of a NE – if at all existing – is an intractable endeavor in the general, agent-based setting.

Mathematically motivated work has focused on more tractable continuum flow problem instances. First to name are the ample variations of the static traffic assignment problem (STA; Patriksson, 2015; Sheffi, 1985). The more recent field of dynamic traffic assignment (DTA) evolved approximately jointly with ATA. DTA still struggles with fundamental model properties like monotonicity and solution uniqueness (Friesz et al., 2021; Han et al., 2019; Wang et al., 2018).

2.2 Problem statement and standard solution approaches

Concrete ATA instances are usually process-based, in that long-term solutions "emerge" rather than being prescribed by an equilibrium-like model formulation. The travel plans of a synthetic traveler population are confronted with a simulation-based representation of the transport infrastructure and its ability to accommodate the desired travel. A typical ATA formulation looks as follows (Nagel and Flötteröd, 2012).

1. Scenario specification.
   (a) Physical representation of the considered scenario: This comprises the road network and possibly the public transport system.
   (b) Synthetic population: The synthetic population is in terms of its socio-demographics representative for the real population of the study region.
   (c) Exogenous travel behavior: The synthetic population is equipped with behavioral features that do not change in the assignment. For instance, car ownership and residence/work/activity locations may be inferred by an upstream travel demand model.
2. Iterative ("day-to-day") computation of a stationary system state.
   (a) All synthetic travelers decide individually how they want to travel in the upcoming day in order to realize their activity schedules. Travelers use their possibly available information about congestion, delay, crowding, … from previous days to select departure times, travel modes, routes.
   (b) All travelers move according to their chosen travel plans through the transport system. Queues and delays may arise at capacity bottlenecks, and not all travelers may experience what they have expected: Delayed travelers may miss public transport connections, time-critical activities (work) may not be reached in time.
   (c) After the simulated day, all travelers process their most recent travel experience. This information serves as a basis for the selection of improved travel plans in the upcoming iteration.

When iterating this process many times, stationary conditions may arise, which may then be considered representative of how the real system under consideration would evolve in the long term. The present work considers a setting where between-iteration variability is increasingly dampened in order to attain an almost (simulation-)noise free point solution. The usual approach to this is still based on variations of the method of successive averages (MSA; Liu et al., 2007; Sheffi, 1985; Blum, 1954), which in the ATA setting amounts to gradually reducing the number of plan-changing travelers per assignment iteration (Ameli et al., 2020; Levin et al., 2015).

3 Method

3.1 Framework

The presentation in this subsection is a very much condensed version of Flötteröd (2022). The method bears similarity with formulations from evolutionary game theory (Sandholm, 2015), the main difference being that it (i) considers a finite population of heterogeneous decision makers and (ii) attempts to identify a point solution rather than a stationary system state.

Consider a population of $n = 1, \dots, N$ travelers. Denote by x_n a travel plan of traveler n and by $x = (x_n)$ the vector of all travel plans in the population. Let $d(x, y)$ be the (yet to be defined) distance between two population plans x and y.

It is assumed that the following takes place in every iteration k of the assignment process: (i) Every traveler n receives a stochastic utility realization representing the performance of that traveler's previously chosen travel plan $x_n^{(k-1)}$. (ii) The traveler then constructs a new candidate travel plan $r_n^{(k)}$, from which it expects a utility increase of $g_n^{(k)}$.

The assignment logic then decides, in every assignment iteration k, which travelers get to switch to their candidate plan. It does so by attempting to minimize a gap function $g^{*(k)} = \sum g_n^{*(k)}$ where $g_n^{*(k)}$ represents the largest possible utility improvement that traveler n may expect from changing its travel plan. An operational formulation is obtained by not explicitly minimizing the gap function but the following upper bound $b^{(k)}$ thereof:

$$g^*(x^{(k+1)}) - g^*(x^{(k)}) \quad \leq \quad b^{(k)}. \tag{1}$$

The concrete approach makes use of the following formulations:

$$x_n^{(k+1)} \quad = \quad \begin{cases} r_n^{(k)} & \text{if } \lambda_n^{(k)} = 1 \\ x_n^{(k)} & \text{if } \lambda_n^{(k)} = 0 \end{cases} \tag{2}$$

$$b^{(k)} \quad = \quad -\sum_n \lambda_n^{(k)} g_n^{(k)} + \theta \cdot d(x^{(k)}, x^{(k+1)}) \quad \text{with } \theta \geq 0. \tag{3}$$

Equation 2 introduces the binary replanning indicators $\lambda_n^{(k)}$,which indicate if traveler n is allowed to switch in iteration k to its candidate plan $r_n^{(k)}$ or maintains its current plan $x_n^{(k)}$. Let $\lambda^{(k)} = (\lambda_n^{(k)})$ be the vector of replanning indicators in the entire traveler population. Equation 3 defines $b^{(k)}$ as a sum of (i) $\sum_n \lambda_n^{(k)} g_n^{(k)}$ representing the expected utility gains of all plan-switching travelers, and (ii) $d(x^{(k)}, x^{(k+1)})$ being the distance between the current population plans and those resulting from replanning. Selecting in every assignment iteration k a $\lambda^{(k)}$ vector of replanning indicators that minimize $b^{(k)}$ hence minimizes an upper bound on the upcoming iteration's gap.

The concrete formulation of Flötteröd (2022) instantiates the function $d(\cdot,\cdot)$ as a Hamming distance, meaning that it merely counts how many travelers switch plans from one iteration to the next. The parameter θ cannot be explicitly determined; it is hence gradually increased over assignment iterations. Since θ functions as a penalty weight on the number of replanners in 3, this effectively leads to a gradual decrease in the number of plan-switching travelers.

Using a Hamming distance and adjusting $\theta^{(k)}$ such the number of plan-switching travelers falls with one over the assignment iteration number almost recovers the "sorting technique" of Sbayti et al. (2007), where in each assignment iteration the the (MSA-controlled share of) travelers with the largest anticipated utility improvement get to switch plans. The methods differ only in that (3) uses expected utility improvements $g_n^{(k)}$, whereas Sbayti et al. (2007) relies on noisy realizations thereof.

The present work improves over the original heuristic of Flötteröd (2022) in two regards: (i) Using a Hamming distance means that the difference between two travel plans is either zero or one, independently of how much they deviate. A more differentiated distance measure between population travel plans is hence developed. (ii) Gradually increasing θ (resp. reducing the replanning rate) requires a reduction schedule, which either needs to be externally given or endogenously constructed. A new endogenous logic is presented.

3.2 Making the framework operational: population distances

The population distance $d(\cdot,\cdot)$ in (3) is instrumental in controlling the amount by which network conditions change from one assignment iteration to the next. To reflect this effect as closely as possible in a distance measure, the travel plans contained in a population are mapped onto the transport network before a distance is constructed. This aims to (coarsely) approximate a full-fledged network loading. For notational simplicity, the following presentation considers only trip-makers (not trip-chain-makers) in a single (not multi-modal) network composed of links and nodes.

Omitting the assignment iteration index k and letting $t \in \mathbb{R}$ be the real-valued within-day time, the within-day time-smoothed inflow to network link i at time t is defined as

$$z_i(t;x) = \frac{\mu}{2c_i}\sum_m e^{-\mu|r_{mi}-t|}x_{mi} \qquad (4)$$

where x is the considered traveler population, c_i is the flow capacity (in vehicles per time unit) of link i, x_{mi} is the zero/one indicator of trip maker m using link i, and r_{mi} is the entry time of trip maker m on link i (undefined for $x_{mi} = 0$). This means that all discrete vehicle entries are represented by a two-sided exponential function that smears their entry out forward and backward in time. $\mu > 0$ defines the rate at which this exponential kernel falls over time. The exponential form is chosen for tractability reasons, which will become clear further below. The normalization with $\mu/2c_i$ is set such that integrating the smoothed flow contribution of a single particle over the entire real line yields $1/c_i$. This implies that smoothing a particle flow at capacity rate c_i over a time interval much larger than $1/\mu$ yields a smoothed flow value of approximately one.

Based on this, the following (square) distance between two traveler populations x and y over all links and the entire real line of within-day times is considered:

$$d^2(x, y) = \sum_i \int_{t=-\infty}^{\infty} (z_i(t; x) - z_i(t; y))^2 \, dt. \quad (5)$$

The exponential form of (4) allows for a closed-form solution to this expression. For this, the similarity $k_i(x_m, x_n)$ of two trip makers m and n on link i is defined as follows:

$$\begin{aligned} k_i(x_m, x_n) &= \int_{t=-\infty}^{\infty} z_i(t, x_m) z_i(t, x_n) dt & (6)\\ &= \frac{\mu^2}{4c_i^2} e^{-\mu|r_{mi} - r_{ni}|} (1/\mu + |r_{mi} - r_{ni}|) x_{mi} x_{ni}. & (7) \end{aligned}$$

Letting

$$\begin{aligned} a_i(x_m, x_n) &= \int_{t=-\infty}^{\infty} (z_i(t, x_m) - z_i(t, x_n))^2 & (8)\\ &= k_i(x_m, x_n) + k_i(x_m, x_n) - 2k_i(x_m, x_n) & (9)\\ a(x_m, x_n) &= \sum_i a_i(x_m, x_n) & (10) \end{aligned}$$

and using (2), one obtains

$$d^2(x^{(k)}, x^{(k+1)}) = \sum_m \sum_n \lambda_m^{(k)} \lambda_n^{(k)} a(x_m^{(k)}, r_n^{(k+1)}). \quad (11)$$

The computational burden of evaluating this expression grows quadratically with the population size. This requires attention because it may impair scalability of the approach for (very) large populations. The following properties work in favor of this scalability: (i) Given that most particles do not enter the same links at nearly the same time, most a_i addends in (9),(10) are close enough to zero to be ignored. (ii) Non-zero a_i values can be efficiently stored per link i, meaning that the quadratic summation effort is reduced to a double sum for each link over all particles ever entering that link, instead of a double sum over all particles in the population. Computational cost may be further reduced by not considering all links (e.g. by accounting for possible linear dependencies of network flows); this possibility is not explored here.

3.3 Making the framework operational: approximate bound minimization

The distance measure (11) is nonlinear, meaning that minimizing the upper gap bound (3) amounts to a nonlinear binary problem. An iterative heuristic along the lines of Merz and Freisleben (2002) is therefore deployed.

Letting

$$b_n^{(k)} = \sum_m \lambda_m^{(k)} a(r_m^{(k)}, x_n^{(k)}), \quad (12)$$

one has

$$d^2(x^{(k)}, x^{(k+1)}) = \sum_n \lambda_n^{(k)} b_n^{(k)}. \tag{13}$$

Now considering a given replanner configuration $\lambda^{(k)} = (\lambda_n^{(k)})$ and the particular traveler m.

- Switching $\lambda_m^{(k)}$ from zero to one requires to add $a(r_m^{(k)}, x_n^{(k)})$ to $b_n^{(k)}$ for all n and changes the population square distance change by $\Delta d^2(x^{(k)}, x^{(k+1)}) = 2b_m^{(k)} + a(x_m^{(k)}, r_m^{(k)})$.

- Switching λ_m from one to zero requires to subtract $a(r_m^{(k)}, x_n^{(k)})$ from $b_n^{(k)}$ for all n and changes the population square distance by $\Delta d^2(x^{(k)}, x^{(k+1)}) = -2b_m^{(k)} + a(x_m^{(k)}, r_m^{(k)})$.

This allows to repeatedly sweep through the population, to evaluate for every individual the effect of switching its λ indicator on the population distance and, through this, on the upper bound (3) one wishes to minimize. Once no further improvement is possible, the heuristic terminates and reports the currently best replanning indicator configuration as an approximate minimizer of the upper gap bound.

3.4 Making the framework operational: Endogenous replanning rate

The gap bound (3) depends on a not explicitly known parameter $\theta \geq 0$. Pursuing a heuristic and operational approach, this parameter is subsequently set to facilitate the efficient approximation of ATA solutions rather than adhering to its strict meaning, which derives from the (unknown) Lipschitz coefficients of the expected utility function used in the behavioral model; details again in Flötteröd (2022).

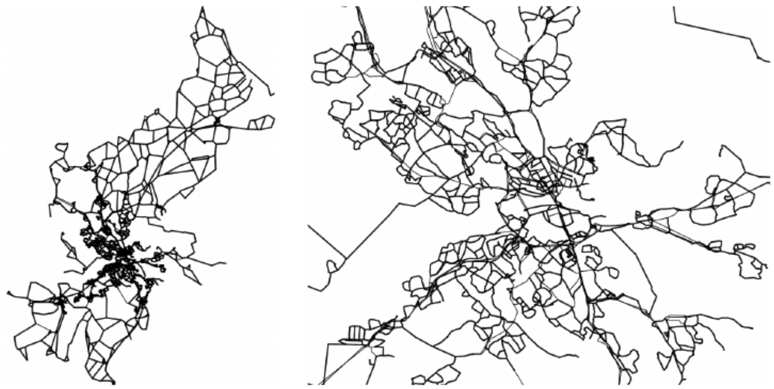
The quantity $g^{(k)} = \sum_n g_n^{(k)}$ is subsequently used as a proxy for the (unknown) population gap $g^{k(*)}$. Let $\eta^{(k)}$ be the anticipated reduction rate of $g^{(k)}$ before assignment iteration k; $\eta^{(k)}$ can be estimated from previous assignment iterations. Postulate (this is heuristic) that a minimization of the current iteration's upper gap bound implies an upcoming gap reduction of at least $\eta^{(k)} g^{(k)}$:

$$-\eta^{(k)} g^{(k)} \stackrel{!}{=} -\sum_n \lambda_n^{(k)} g_n^{(k)} + \theta^{(k)} d(x^{(k)}, x^{(k+1)}) \tag{14}$$

$$\Leftrightarrow \quad \theta^{(k)} = \frac{\sum_n \lambda_n^{(k)} g_n^{(k)} - \eta^{(k)} g^{(k)}}{d(x^{(k)}, x^{(k+1)})}. \tag{15}$$

However, when minimizing the upper gap bound, the solution $\lambda^{(k)}$ is not yet known, so neither is $\theta^{(k)}$. Section 3.3's iterative heuristic for tackling the nonlinear binary replanner selection problem is hence complemented with a logic that estimates $\theta^{(k)}$ along these iterations. One iteration consists of the following two steps:

1. Use (15) to compute $\theta^{(k)}$ given the current replanner selection $\lambda^{(k)}$.



(a) entire network (b) central network

Figure 1: Stockholm network

2. Update $\lambda^{(k)}$ to reduce the upper gap bound (3) given the current value of $\theta^{(k)}$.

It can be shown that if the replanner selection is implemented such that an upper bound reduction of at least $\varepsilon > 0$ is achieved in every iteration of Section 3.3's heuristic, then the θ sequence increases monotonically. Unless terminated because no improvement of at least ε is possible, a θ value will hence eventually be reached that is so large that all replanning ceases, in which case an all-zero λ vector is returned. Within each assignment iteration, the heuristic replanner selection logic with an adaptive θ coefficient is hence ensured to terminate after a finite number of replanner switches.

4 Results and tentative conclusion

The experimental setup presented here coincides largely with that of Flötteröd (2022) and is presented using the same wording. The results are, however, new.

As a case study, the Greater Stockholm region in Sweden is considered. Figures 1(a) and (b) display the network. Only car traffic is considered; a 24-hour dynamic network loading is computed by a queueing simulation that approximates a particle-discrete kinematic wave model. The conflict resolution logic of its intersection model is stochastic. Travel behavioral degrees of freedom are route and departure time choice. Proposal routes are generated by computing time-dependent shortest paths against the previous assignment iteration; proposal departure times are generated by random variations.

The following ATA heuristics are considered.

ID "Identically distributed". Given a replanning rate λ, each traveler switches to its proposal plan with identical probability λ.

Sbayti The method of Sbayti et al. (2007), where in each assignment iteration the λ-share of travelers with the largest anticipated utility improvements are allowed to switch plans.

UB "Upper bound". The proposed assignment heuristic.

These methods are organized in the columns of Figure 2. All algorithms are evaluated for three different congestion levels: *low*, *medium* (the result of an approximate model calibration), and *high*. The congestion levels are organized in the rows of the following figures. All combinations of algorithms and congestion levels are evaluated for two different replanning rate schedules: *MSA* (λ falls with one over the iteration number) and *sqrtMSA* (λ falls with one over the square root of the iteration number).The following trends can be observed: Gap levels increase with congestion levels. The *MSA* step size rule performs systematically unfavorably compared to the *sqrtMSA* rule. *Sbayti* reaches lower gaps than *ID*. As congestion increases, the proposed *UB* method increasingly outperforms *Sbayti*.

In light of these results, it can be tentatively concluded that the proposed method delivers an at least competitive, if not superior, performance in approximating solutions to the ATA problem. Being derived from first principles, the proposed method is conceptually more involved than alternative heuristics. The method also exhibits a somewhat higher computing time per assignment iteration, which, however, is counteracted by its ability to attain the same level of ATA precision as competing techniques with a lower number of assignment iterations. Important for its applicability, the method requires no externally set parameters, not even an MSA-like step size rule.

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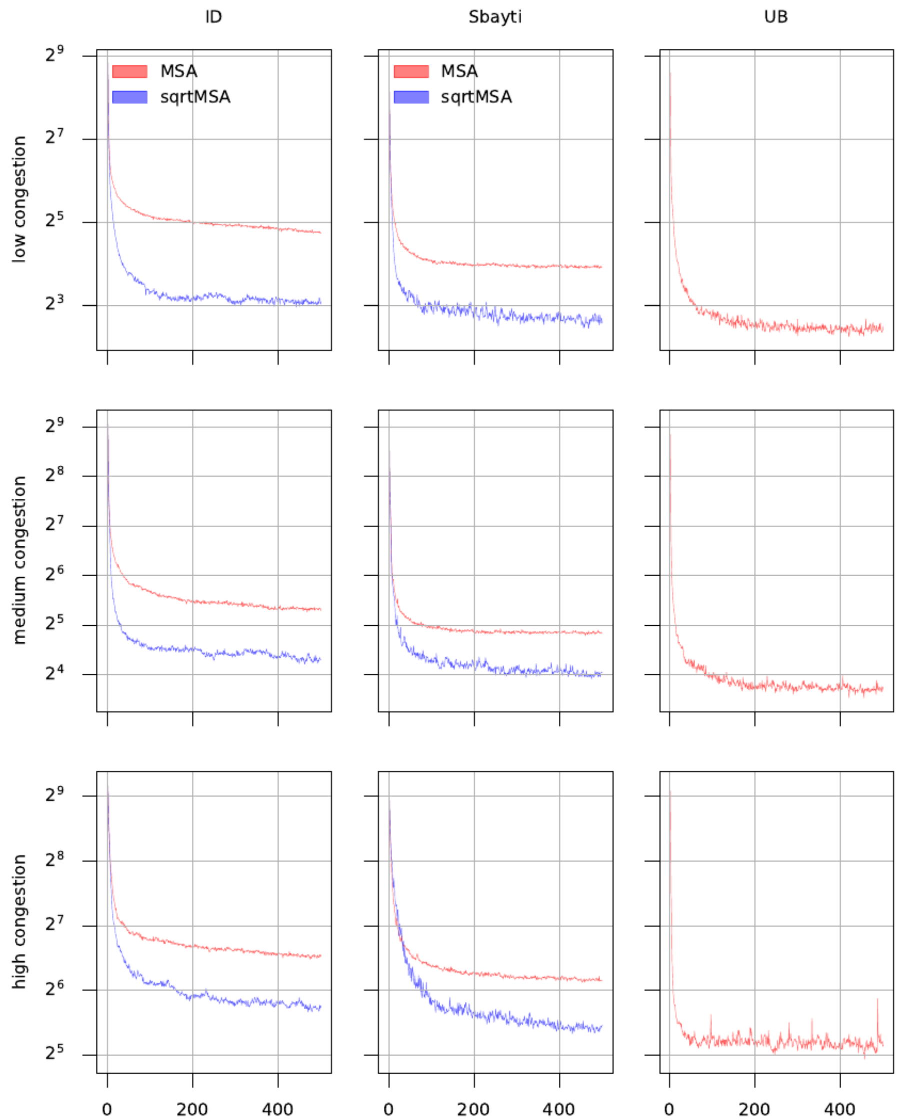

Figure 2: Gap over assignment iteration

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