How different charging prices can contribute to the balancing of ride-hailing services with electric vehicles

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SHORT SUMMARY

Motivated by the growing popularity of both the services offered by the ride-hailing companies and the electric vehicles (EVs), we study a scenario in which a central body e.g., the government, wants to influence how the EVs of different ride-hailing companies spread among different charging stations by offering discounted prices of charging. Because the companies share the charging infrastructure, they inherently compete to minimize their expected total queuing times at the charging stations. From the perspective of Stackelberg and Inverse Stackelberg games we analyze two pricing mechanisms for the government that guarantee existence of a Nash equilibrium of the game played between the ride-hailing companies. We compare their performance in a case study based on taxi data from the city of Shenzhen and also show how the systems behave in terms of robustness.

Keywords: Stackelberg game, Inverse Stackelberg game, Electric vehicle charging

1. INTRODUCTION

In the recent years, the increase in affordability of alternative fuel vehicles and their prospective positive long term effects on the environment have caused the number of electric vehicles (EVs) on the streets to climb steeply (Internationa Energy Agency, 2021). Given that nowadays the ride-hailing companies constitute the essential part of the transportation services offered within a city, the electric vehicles could soon become the central part of the fleet that a ride-hailing company manages. As a consequence, the charging management of the vehicles is likely to become a significant part of the company's operation. Even today the ride-hailing companies offer access to different service facilities to their drivers so it is not unlikely that they would also offer discounted charging to incentivise the drivers to follow the company's management desires. For the company, the benefit of introducing the pricing incentives would be twofold. Not only could it increase the coverage by sending vehicles to charge in the areas where there is demand, but at the same time it could maximize the availability of the fleet by incentivising the drivers to charge before the demand peaks. Moreover, taking into account the asymmetric distribution of origins and destinations, the pricing incentives could contribute in re-balancing vehicles in regions with higher demand.

Inspired by this vision, we study a scenario in which a central body, e.g., the government, has a desire about how the vehicles should spread out among different charging stations in a region where the operators of the several ride-hailing companies try to minimize their operational cost. Since the companies utilize the shared charging infrastructure, they are interested in directing their vehicles to different charging stations so as to minimize the queuing time at the stations. Our goal in this paper is to compare designs of intelligent pricing mechanisms that stem from the game theoretic analysis of the problem. There is an extensive literature applying game theoretic models based on congestion, mean-field, Stackelberg and Inverse Stackelberg games to solve different problems in the domain of smart mobility systems (J. Zhang et al., 2018; Brown & Marden, 2020; L. Zhang et al., 2019; Paccagnan et al., 2016, 2019; Ma et al., 2013; Tushar et al., 2012; Laha et al., 2019; Basar & Srikant, 2002; Groot et al., 2012; Staňková et al., 2011). Our model lies at the intersection of aggregative games primarily used in (Paccagnan et al., 2016, 2019; Ma et al., 2013) and the inherent leader-follower structure present in the Stackelberg and Inverse Stackelberg games utilized in (Tushar et al., 2012; Laha et al., 2019; Basar & Srikant, 2002; Groot et al., 2012; Staňková et al., 2011). However, it differs from the previously mentioned Stackelberg/Inverse Stackelberg setups in a sense that the leader, i.e., the government is not a revenue maximizing player but rather tries to achieve predefined total vehicle accumulations at different charging stations in order to help fight congestion in the area.

This paper is the continuation of the work presented in (Maljkovic, Nilsson, & Geroliminis, 2021) where we originally defined the problem structure and showed that it is possible to construct pricing policies in an Inverse Stackelberg manner such that the Nash-equilibrium of the game played between the companies follows the central authority's desire. Moreover, we show that it can be computed in a decentralized way, with little private information exchange between the ride-hailing companies and the government and under the reachability constraints imposed by the battery state of the vehicles in individual car fleets. The pricing mechanism based on the Inverse Stackelberg game allows more freedom as different charging prices are calculated for each individual ride-hailing company allowing for perfect Nash equilibrium placement under the reachability constraints. In this paper, we present a more realistic case study based on the city of Shenzhen in which we compare the performance of the originally proposed model to the performance of the Stackelberg model that assumes a fixed pricing vector is announced by the government. In general, computing the optimal decision of the leader, i.e., the optimal pricing vector issued by the government, is NP hard (Basilico et al., 2020; Coniglio et al., 2020) for one-leader multiple-followers structures which generally motivates the usage of pricing policies. Because we assume that the reachability constraints of the companies should be kept private, i.e., the exact locations of the vehicles and their battery levels should be stored locally, the algorithms for computing the approximate Nash equilibrium in a Stackelberg game that rely on each player having the perfect information such as in (Wang et al., 2021) are not applicable. Instead, we test the performance of a Stackelberg model with the pricing vector obtained through extensive grid search over the space of reasonable charging prices on multiple scales. This setup models a scenario in which the government tries to model the behaviour of the ride-hailing companies based on the historical data which helps it establish suitable charging prices. Finally, we show that on average the pricing mechanism based on the Inverse Stckelberg game performs better than the fixed pricing for all companies in terms of robustness.

The paper is outlined as follows: the rest of this section is devoted to introducing some basic notation. In Section 2 we revise the structure of the model, introduce the Stackelberg mechanism, re-introduce the original pricing mechanism and state the main results from (Maljkovic et al., 2021) relevant for further analysis. In the following section, Section 3 we compare the performance of the two pricing mechanisms through extensive simulations of a new case study based on the network topology of the city of Shenzhen. The final section contains the concluding remarks and some ideas for future research.

Notation

We let $\mathbb{R}$ denote the set of real numbers, and $\mathbb{R}_+$ the set of non-negative reals. We let $\mathbf{0}_m$ and $\mathbf{1}_m$ denote the all zero and all one vector of length m respectively, and $\mathbb{I}_m$ the identity matrix of size $m \times m$. For a finite set $\mathcal{A}$, we let $\mathbb{R}^{\mathcal{A}}_{(+)}$ denote the set of (non-negative) real vectors indexed by the elements of $\mathcal{A}$, $\mathbb{Z}^{\mathcal{A}}_{(+)}$ denote the set of (non-negative) integer vectors indexed by the elements of $\mathcal{A}$,

$|\mathcal{A}|$ the cardinality of $\mathcal{A}$ and we let $\mathcal{P}_{\mathcal{A}}$ be the probability space over the set, i.e. $\mathcal{P}_{\mathcal{A}} := \{x \in \mathbb{R}_+^{\mathcal{A}} \mid \sum_{i\in\mathcal{A}} x_i = 1\}$. For a diagonal matrix $A \in \mathbb{R}^{n\times n}$, we let A^* denote its pseudo-inverse, i.e.,

$$A_{ii}^* := \begin{cases} 1/A_{ii} & \text{if } A_{ii} > 0, \\ 0 & \text{otherwise,} \end{cases} \quad 1 \le i \le n\,.$$

2. METHODOLOGY

Model

We begin this section by recalling the system model and the parameters that describe it. We consider a setting where different ride-hailing companies have access to common charging stations for their electric vehicles. We let $\mathcal{C}$ denote the set of companies, and $N_i > 0$ the fleet size of the company $i \in \mathcal{C}$. The vector of the number of vehicles for all companies is denoted by $N \in \mathbb{R}_+^{\mathcal{C}}$. We let $\mathcal{M}$ represent the set of charging stations, and $M_j > 0$ the number of spots available at each charging station $j \in \mathcal{M}$, i.e., the charging station's capacity. The vector of all charging station capacities is denoted by $M \in \mathbb{R}_+^{\mathcal{M}}$ and the cardinality of $\mathcal{M}$ as $m = |\mathcal{M}|$.

For each company $i \in \mathcal{C}$, we let $\mathcal{V}_i$ be the set of its vehicles with $|\mathcal{V}_i| = N_i$. Let $x^i \in \mathcal{P}_{\mathcal{M}}$ denote the fraction of vehicles that the company wants to send to each charging station, i.e., x_j^i is the fraction of vehicles from company $i \in \mathcal{C}$ that will be sent to charging station $j \in \mathcal{M}$. Furthermore, let $n_j^i \in \mathbb{Z}_+$ denote the integer number of vehicles, associated with the continuous allocation x_j^i, that the operator of the fleet would send to station j. Since not all charging stations are reachable for all vehicles and hence not all choices of x^i are feasible, we define for each company the feasibility sets $\mathcal{F}_j^i := \{v \in \mathcal{V}_i \mid v \text{ can reach station } j\}$. We say that a continuous allocation vector x^i is feasible, if it allows the operator of the company to choose any discrete allocation $n^i \in \mathbb{Z}_+^{\mathcal{M}}$ where individual n_j^i can be either $\left\lfloor N_i x_j^i \right\rfloor$ or $\left\lceil N_i x_j^i \right\rceil$ under the constraints that $\sum_{j\in\mathcal{M}} n_j^i = N_i$ and that there exists a feasible matching between the vehicles and the charging stations for the chosen n^i.

For each company $i \in \mathcal{C}$, we let $\mathcal{K}_i \subseteq \mathcal{P}_{\mathcal{M}}$ denote the set of all feasible x^i. Furthermore, we define $\mathcal{K} := \prod_{i\in\mathcal{C}} \mathcal{K}_i$ and $\mathcal{K}_{-i} := \prod_{j\in\mathcal{C}\setminus i} \mathcal{K}_j$. We let $x := \left[x^i\right]_{i\in\mathcal{C}} \in \mathcal{K}$ denote all companies' decision vectors, $x^{-i} := \left[x^j\right]_{j\in\mathcal{C}\setminus i} \in \mathcal{K}_{-i}$ denote the decision vectors of all companies except the company i, $\sigma(x) := \sum_{i\in\mathcal{C}} N_i x^i \in \mathbb{R}_+^{\mathcal{M}}$ denote the vector consisting of the total number of vehicles that have chosen each station and $\sigma\left(x^{-i}\right) := \sum_{j\in\mathcal{C}\setminus i} N_j x^j \in \mathbb{R}_+^{\mathcal{M}}$ denote the vector consisting of the number of vehicles from the other companies that have chosen each station.

The model that we consider allows the government to be interested in balancing the vehicles so as to minimize the personal objective of the form:

$$\min_{\sigma(x)} J_G(\sigma(x)) = \min_{\sigma(x)} \frac{1}{2}\sigma(x)^T A_G \sigma(x) + b_G^T \sigma(x)\,, \tag{1}$$

for some diagonal matrix $A_G \succ 0$ and $b_G \in \mathbb{R}^{\mathcal{M}}$. We are interested in minimizing a special case of (1) that corresponds to balancing the vehicles so that the vector of numbers of vehicles charging at each station equals $\hat{N} \in \mathbb{R}_+^{\mathcal{M}}$, i.e., to minimize

$$J_G(\sigma(x)) = \frac{1}{2}\|\sigma(x) - \widehat{N}\|_{2,A_G}^2\,, \tag{2}$$

where $A_G \succ 0$ gives the government the possibility to penalize deviations from the desired number of vehicles differently at different stations. It should be noted that minimizing (2) corresponds to minimizing (1) when $b_G = -A_G\widehat{N} \in \mathbb{R}^{\mathcal{M}}$.

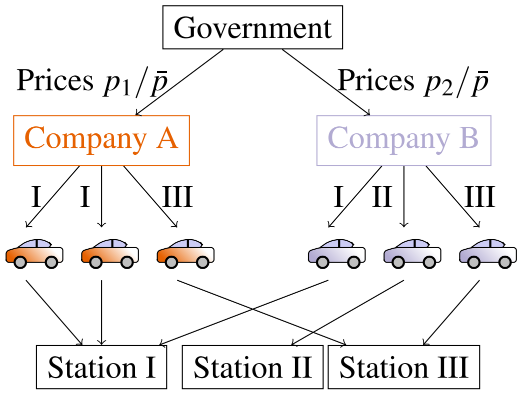

Figure 1: Schematic sketch of the problem setting. The government wants to balance the vehicle load on different charging stations either through dynamics pricing policies p_1 and p_2 or through optimal static pricing vector $\bar{p}$.

Based on the pricing mechanism that we analyze, in order to steer the companies decisions towards the minimizer of (1), the government will either announce an individual pricing policy for each company $i \in \mathcal{C}$ given by $p_i\left(x^i, x^{-i}\right) : \mathcal{K}_i \times \mathcal{K}_{-i} \to \mathbb{R}^{\mathcal{M}}$, or a fixed pricing vector $\bar{p} \in \mathbb{R}^{\mathcal{M}}$:

$$\pi_i\left(x^i, x^{-i}\right) = \begin{cases} p_i\left(x^i, x^{-i}\right), & \text{Inverse Stackelberg game} \\ \bar{p}, & \text{Stackelberg game} \end{cases}$$

After the pricing policies are announced, the government and the companies play a game in which every company is trying to minimize its own operational cost, under the constraint that all the company's vehicles must be able to reach a charging station and that no vehicle is assigned to multiple charging stations in order to respect the optimal continuous assignment. We model the operational cost for each company as a sum of three terms:

$$J^i\left(x^i, x^{-i}\right) = \underbrace{\frac{1}{2}\left(x^i\right)^T A_i x^i + \left(x^i\right)^T B_i \sigma\left(x^{-i}\right) + c_i^T x^i}_{J_1^i\left(x^i, \sigma(x)\right)} + \underbrace{\left(x^i\right)^T D_i \pi_i\left(x^i, x^{-i}\right)}_{J_2^i\left(x^i, \pi_i\left(x^i, x^{-i}\right)\right)} + \underbrace{f_i^T x^i}_{J_3^i\left(x^i\right)}, \tag{3}$$

where $J_1^i\left(x^i, \sigma(x)\right)$ denotes the expected queuing cost, $J_2^i\left(x^i, \pi_i\left(x^i, x^{-i}\right)\right)$ is the expected charging cost and $J_3^i\left(x^i\right)$ is the negative expected revenue. $D_i \in \mathbb{R}^{\mathcal{M}\times\mathcal{M}}$ is diagonal, $D_i \succ 0$ and the entries $\left(D_i\right)_{kk}$ can be interpreted as the portion of the total charging demand of the i-th company to be served at the charging station k. We model the expected queuing cost as

$$J_1^i\left(x^i, \sigma(x)\right) = N_i\left(x^i\right)^T Q\left(\sigma(x) - M\right), \tag{4}$$

and the negative expected revenue as

$$J_3^i\left(x^i\right) = \left(e_i^{\text{arr}}\right)^T N_i x^i - \left(e_i^{\text{pro}}\right)^T N_i x^i, \tag{5}$$

where $Q \in \mathbb{R}^{\mathcal{M}\times\mathcal{M}}$ is a positive definite diagonal scaling matrix whose diagonal entries describe how expansive it is to queue in the regions around charging stations, $e_i^{\text{arr}} \in \mathbb{R}^{\mathcal{M}}$ is the average cost of a vehicle being unoccupied while traveling to a charging station and the vector $e_i^{\text{pro}} \in \mathbb{R}^{\mathcal{M}}$ denotes the expected profit in regions around different charging stations estimated from historical data. We include e_i^{pro} in the analysis because a company might decide to pay more for charging if in the region around the station there is higher demand. Substituting (4) and (5) into (3) yields the following values of the parameters:

$$A_i := 2N_i^2 Q,\ B_i := N_i Q,\ c_i := -N_i Q M,\ f_i := N_i\left(e^{\text{arr}} - e^{\text{pro}}\right). \tag{6}$$

The operator of the company $i \in \mathcal{C}$ would like to allocate the EVs according to:

$$x^{i*} \in \underset{x^i \in \mathcal{K}_i}{\arg\min}\, J^i\left(x^i, x^{-i}\right). \tag{7}$$

We say that the government and the companies admit a system optimum if there exists x^* that minimizes (1) and satisfies (7) for all $i \in \mathcal{C}$. We showed in (Maljkovic et al., 2021) that if we can reduce the decision space of the companies to convex subsets $\overline{\mathcal{K}}_i \subseteq \mathcal{K}_i$, the proposed Inverse Stackelberg game based pricing mechanism yields a unique system optimum. In the following subsection we extend this result to Stackelberg games that govern the idea of having a single pricing vector applied to all companies. The problem that we address is depicted in Figure 1 and regardless of the pricing mechanism, can be formally stated as follows:

Problem 1 *Design pricing $\pi_i\left(x^i, x^{-i}\right)$ and the constraint sets $\overline{\mathcal{K}}_i \subseteq \mathcal{K}_i$ such that there is a unique Nash equilibrium of the game G defined as*

$$G := \left\{ \min_{x^i \in \overline{\mathcal{K}}_i} J^i\left(x^i, x^{-i}\right), \forall i \in \mathcal{C} \right\}, \tag{8}$$

with $J^i\left(x^i, x^{-i}\right)$ defined as in (3). Moreover, the Nash equilibrium should be such that it also minimizes the government cost $J_G(x)$ in (1) and the design of the constraint sets $\overline{\mathcal{K}}_i$ such that existence of a feasible discrete allocation scheme for each company is guaranteed.

Theoretical preliminaries

We start this subsection by recalling from (Maljkovic et al., 2021, P.1) that it is possible to analytically construct convex sets $\overline{\mathcal{K}}_i \subseteq \mathcal{K}_i$ based on the feasibility sets $\mathcal{F}^i_j$ and the condition of the Hall's marriage theorem for bipartite graphs, that guarantee feasibility of x^i defined as in Section 2.1. Sets $\overline{\mathcal{K}}_i$ reflect the current state of the car fleets and in realistic scenarios are private, i.e., not known to the government, as they encompass information about the true location of the vehicles and their current and desired battery level. We let $\overline{\mathcal{K}} := \prod_{i \in \mathcal{C}} \overline{\mathcal{K}}_i$ and $\overline{\mathcal{K}}_{-i} := \prod_{j \in \mathcal{C} \setminus i} \overline{\mathcal{K}}_j$ and show that for these sets the Stackelberg game based model also gives raise to a unique Nash equilibrium.

Theorem 1 *For all companies $i \in \mathcal{C}$, let the sets $\overline{\mathcal{K}}_i$ be designed as in (Maljkovic et al., 2021, P.1). Furthermore, let A_i, B_i, c_i, and f_i be defined as (6). Then, with the pricing mechanism $\pi_i\left(x^i, x^{-i}\right) := \bar{p} \in \mathcal{R}^{\mathcal{M}}$, the game G in (8) has a unique Nash equilibrium.*

Proving existence and uniqueness of the Nash equilibrium follows exactly the same line of thought as in (Maljkovic et al., 2021, T.1). Existence is guaranteed since $A_i = 2N_i^2 Q \succ 0$, while uniqueness is guaranteed since for all $i \in \mathcal{C}$, $B_i \succ 0$ and Γ can be partitioned into $N \times N$ blocks $\Gamma_{ij} \in \mathcal{R}^{\mathcal{M} \times \mathcal{M}}$:

$$\Gamma_{ij} = \begin{cases} -2A_i, & i = j \\ -(B_i + B_j), & i \neq j \end{cases}.$$

For clarity, we also recall the system optimal pricing policies for the Inverse Stackelberg game setup (Maljkovic et al., 2021, D.1) defined for each company $i \in \mathcal{C}$ as:

$$p_i\left(x^i, x^{-i}\right) = D_i^* \left[\frac{1}{2} \overline{\mathrm{A}}_i x^i + \overline{\mathrm{B}}_i \sigma\left(x^{-i}\right) + \Delta_i \right],$$

where $\overline{\mathrm{A}}_i = N_i^2 A_G - A_i$, $\overline{\mathrm{B}}_i = N_i A_G - B_i$ and $\Delta_i = N_i b_G - c_i - f_i$. In order to compute the optimal pricing vector $\bar{p} \in \overline{\mathcal{P}} \subseteq \mathbb{R}^{\mathcal{M}}$ for the Stackelberg based mechanism, we would in general have to solve

$$\bar{p}^* = \arg\min_{p \in \overline{\mathcal{P}}} \frac{1}{2} \sigma\left(x^*\left(\bar{p}\right)\right)^T A_G \sigma\left(x^*\left(\bar{p}\right)\right) + b_G^T \sigma\left(x^*\left(\bar{p}\right)\right),$$

where $x^*\left(\bar{p}\right) \in \overline{\mathcal{K}}$ is the Nash equilibrium of the game (8) for a particular fixed price choice $\bar{p}$. Unfortunately, computing $x^*\left(\bar{p}\right) \in \mathcal{K}$ in closed form is in general not possible and because the

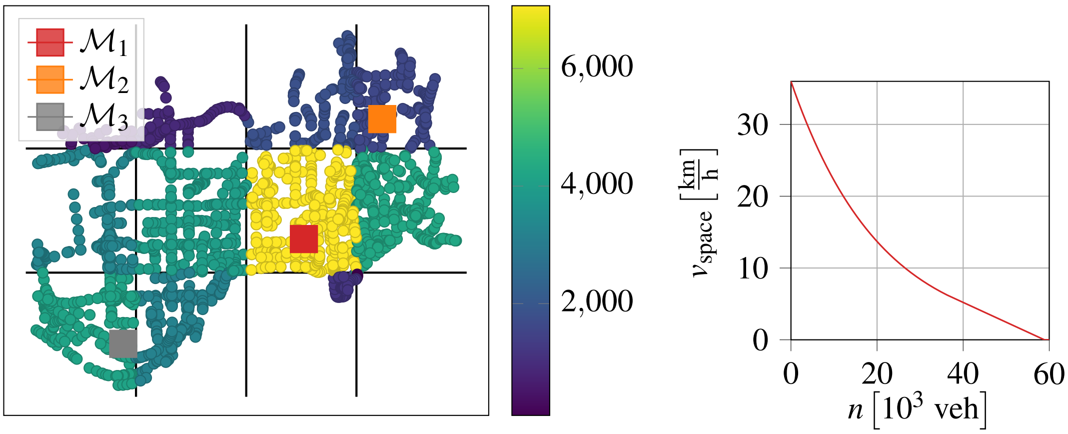

Figure 2: Map of Shenzhen - The network topology used in the case study consists of 1858 intersections connected by 2013 road segments divided in 12 rectangular regions. Color of the nodes within each region indicates the total number of ride-hailing requests whose origin is in that region.

sets $\overline{\mathcal{K}}_i$ are private, the algorithms for computing the approximate Stackelberg equilibrium such as (Wang et al., 2021) that rely on perfect information are not applicable. Instead, in order to approximate the optimal $\bar{p}$, we choose a fixed maximum price $p_{\max} \in \mathbb{R}_+$, set $\overline{\mathcal{P}} = [0, p_{\max}]^{\mathcal{M}}$ and perform $N_{\text{grid}}^{|\mathcal{M}|}$ simulations where we measure $J_G(\sigma(x^*))$ in order to determine what is the best approximation for the given hyper-parameters $p_{\max}$ and N_{grid}, where N_{grid} defines the resolution with which we equidistantly sample $\overline{\mathcal{P}}$ along every axis.

3. RESULTS AND DISCUSSION

Case study

We begin this section by describing the case study in which the two pricing mechanisms can be used to balance the EVs so that the number of vehicles charging at different stations is as close as possible to vector $\hat{N} \in \mathbb{R}^{\mathcal{M}}$. We consider a scenario where 3 ride-hailing companies $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}$, whose fleet sizes are given by $N = [100, 250, 180]^T$, operate in the city of Shenzhen where 3 charging stations $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3\}$ are available. The stations are described by the vector of their capacities $M = [10, 60, 40]^T$ and are located in the areas with different demands for ride-hailing services as shown on the left in Figure 2. We consider a 3 hour long simulation that should represent one of the two peak hour periods during the day. New passengers constantly arrive in the system and either increase the number of private vehicles in the system or request a ride-hailing vehicle to be assigned to them. The space mean speed of vehicles is modelled as a function of the total vehicle accumulation n and according to the network Macroscopic Fundamental Diagram (MFD) (Geroliminis & Daganzo, 2008) obtained from (Ji, Luo, & Geroliminis, 2014). Under the assumption of homogeneous congestion in the city the MFD relation is given by:

$$v_{\text{space}}(n) = \begin{cases} 36\exp\left(-\frac{29n}{600000}\right), & \text{if } n \leq 36000 \\ 6.31 - 0.28\left(\frac{n}{1000} - 36\right), & \text{if } 36000 < n \leq 60000 \\ 0, & \text{if } n > 60000 \end{cases}$$

and is shown in Figure 2 on the right. For simplicity, we assume a linear battery discharge model:

$$s^{\text{curr}}(t + \Delta T) = s^{\text{curr}}(t) - \frac{100}{d_{\max}} v_{\text{space}} \Delta T\,,$$

Table 1: Performance comparison between the two pricing mechanisms

	Station $\mathcal{M}_1$		Station $\mathcal{M}_2$		Station $\mathcal{M}_3$	
	$\bar{p}_1$	$p_i^1\left(x^i, x^{-i}\right)$	$\bar{p}_2$	$p_i^2\left(x^i, x^{-i}\right)$	$\bar{p}_3$	$p_i^3\left(x^i, x^{-i}\right)$
$\mathcal{C}_1$		4.47		2.28		3.10
$\mathcal{C}_2$	10.0	4.54	7.50	2.22	8.50	3.02
$\mathcal{C}_3$		4.69		2.24		3.08
$\hat{N}$	18		111		75	
$\sigma\left(x^*\right)$	17.94	18.01	121.41	110.95	64.64	75.03

where $d^{\max}$ is the max range of the vehicle. We assume that each vehicle starts the simulation with a battery level chosen randomly between 90% and 95% and that after 3 hours of operation, the ride-hailing vehicles whose battery level dropped below 55% are considered for charging. Vector $\hat{N}$ is determined such that the vehicles to be charged are split among the stations in the ratio 1:6:4. The state of each vehicle $v_j \in V_i$ to be charged is described by a tuple $\left(s_j^{\text{curr}}, s_j^{\text{des}}, d_j^{\max}\right)$ where s_j^{curr} represents the current battery level in percentage and s_j^{des} is set to 100%. A charging station k is considered to be feasible for vehicle j if j can reach it with the current battery status, i.e., if $s_j^{\text{curr}} - \frac{100}{d_j^{\max}} d_{j,k} > 0$ where $d_{j,k}$ denotes the shortest path between the vehicle j and the charging station k. The average charging cost is modelled by setting $D_i := N_i R_i$ where diagonal matrix $R_i \in \mathbb{R}^{3\times 3}$ captures the average charging demand per vehicle when choosing each of the charging stations. For infeasible charging stations the average demand is set to 0. If the charging station k is feasible to vehicle $v_l \in V_i$, vehicle's charging demand if k is chosen for charging is defined as $\delta_{l,k} = \beta_l \left(s_l^{\text{des}} - \left(s_l^{\text{start}} - \frac{100}{d_l^{\max}} d_{l,k}\right)\right)$. Here $\beta_l \in \mathbb{R}$ is a scaling coefficient that tells how many units of charge corresponds to 1% of the vehicle's battery. The diagonal element of R_i that corresponds to station k is then given by:

$$(R_i)_{kk} = \left[\frac{1}{\left|\mathcal{F}_k^i\right|} \sum_{l: v_l \in \mathcal{F}_k^i} \delta_{l,k}\right].$$

Regarding the parameters used to describe the negative expected revenue, if station k is infeasible for company $i \in \mathcal{C}$, then we set $\left(e_i^{\text{arr}}\right)_k = 0$, otherwise it is equal to:

$$\left(e_i^{\text{arr}}\right)_k = u_i \cdot P_k \cdot \left[\frac{1}{\left|\mathcal{F}_k^i\right|} \sum_{l: v_l \in \mathcal{F}_k^i} d_{l,k}\right],$$

where $u_i \in \mathbb{R}$ is the monetary value of a vehicle being occupied while driving for 1 km, given in [\$/km] and P_k is the probability of a vehicle being occupied in the region around charging station k. The vector $e_i^{\text{pro}} \in \mathbb{R}^3$ we set to $e^{\text{pro}} = [270.0, 142.0, 201.0]^T$. We fix other parameters to $\beta_l = 1.0$, $Q = 0.1 \cdot \operatorname{diag}(5,1,3)$ and $A_G = 3Q$, vector of probabilities of being occupied $P = [0.35, 0.1, 0.2]$, $u_i = 1.0, \forall i \in \mathcal{C}$ and set the number of iterations for the algorithm to $k = 3000$.

System performance: Stackelberg vs. Inverse Stackelberg

Let us first denote $\bar{p} = [\bar{p}_1, \bar{p}_2, \bar{p}_3]^T \in \bar{\mathcal{P}}$. We performed the grid search for two different price scales $p_{\max} \in \{5.0, 10.0\}$ with the spatial resolution of $N_{\text{grid}} = 20$ points per each axis. In total, 204 vehicles required charging after the 3 hour simulation has been completed and the minimum $J_G\left(\sigma\left(x^*\left(\bar{p}\right)\right)\right) = 64.57$ was obtained for $p_{\max} = 10.0$ and $\bar{p} = [10.0, 7.5, 8.5]^T$. On the other hand, the Inverse Stackelberg mechanism attained the value $J_G\left(\sigma\left(x^*\right)\right) = 0.001$. Complete performance comparison is presented in Table 1 where we give for each charging station the desired vehicle

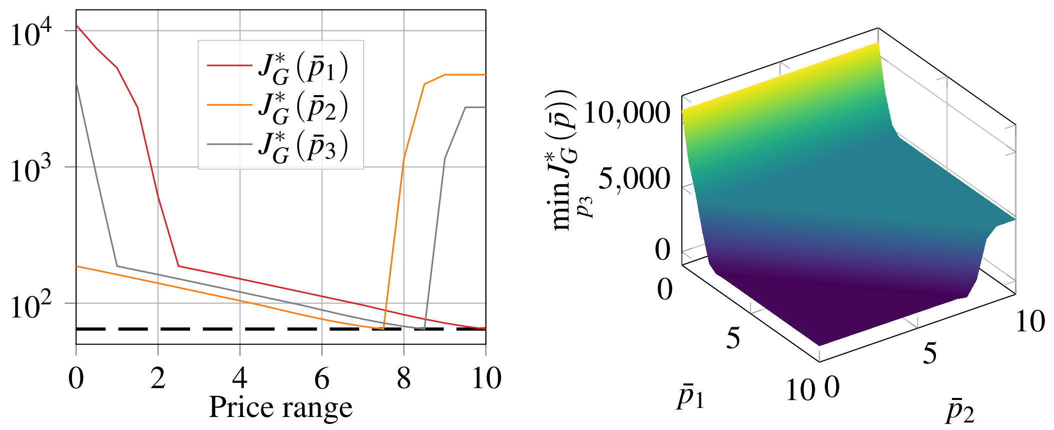

Figure 3: Stackelberg game mechanism. With a slight abuse of notation $J_G^*(\bar{p}) = J_G(\sigma(x^*(\bar{p})))$ and $J_G^*(\bar{p}_i) = \min_{p_j,p_k:j,k\neq i} J_G(\sigma(x^*(\bar{p})))$, the left plot shows what is the minimum value of J_G for a particular price at a particular charging station, e.g., the red line shows what is the minimum value of J_G for a fixed $\bar{p}_1$ over all possible $\bar{p}_2$ and $\bar{p}_3$. The plot on the right shows the minimum of J_G in 3D and indicates which part of the $(\bar{p}_1, \bar{p}_2)$ plane yields significantly higher values of J_G regardless of $\bar{p}_3$.

accumulation $\hat{N}$, attained vehicle accumulation in the Nash equilibrium $\sigma(x^*)$ for both charging mechanisms, fixed price obtained through grid search and used in the Stackelberg setup and the prices of charging for each company in the case of Inverse Stackelberg setup. In this case $\hat{N} = [18, 111, 75]$ and we observe that both mechanisms almost perfectly match the desired accumulation at station $\mathcal{M}_1$. However, the Stackelberg based mechanism fails to balance the vehicles among $\mathcal{M}_2$ and $\mathcal{M}_3$ in a desired manner. Though the better performance of the Inverse Stackelberg based mechanism comes at the expense of utilizing different charging prices for different ride-hailing companies and the risk of having unfair prices between the companies, Table 1 suggests that the gap between the prices is tolerable. As expected, the prices of charging at $\mathcal{M}_1$ and $\mathcal{M}_2$ are in both cases the highest and the lowest respectively, since these are the regions with the highest and the lowest expected profit and the lowest and the highest desired accumulations.

Robustness analysis

Apart from R_i and e^{arr}, all other parameters are inherently known to the government as they characterize the region in which the companies operate. Hence, the government optimum is attainable if the companies are willing to share R_i and e^{arr} that encompass the information about the average charging demand and position of the company's fleet. R_i is considered a more sensitive piece of information compared to e^{arr} as the latter one only depends on the vehicles' distances to charging stations. Hence, we test robustness of the proposed pricing mechanism and show how the system behaves in the same scenario when the government has only an estimate $\overline{D}_i$ of the average charging demand $D_i := N_i R_i$. For a feasible station k, we let $(\overline{D}_i)_{kk} = (D_i)_{kk} + w_k$ where w_k is a noise sample drawn from a normal distribution such that $w_k \sim \mathcal{N}\left(0, (\alpha D_{\min}/4)^2\right)$ with $D_{\min}$ being the minimal, non-zero, diagonal element of D_i. For every α we sample w_k 50 times and report the mean value of the government's loss in the Nash equilibrium. Figure 4 shows that for moderate discrepancies $(\alpha < 0.2)$ between the true and the estimated value of R_i, the attained Nash equilibrium in Inverse Stackelberg game is close to the government's optimum. More importantly, Figure 4 shows that for all magnitudes of perturbation α, the Inverse Stackelberg game based mechanism significantly outperforms the Stackelberg based one in terms of robustness with respect to parameter D_i.

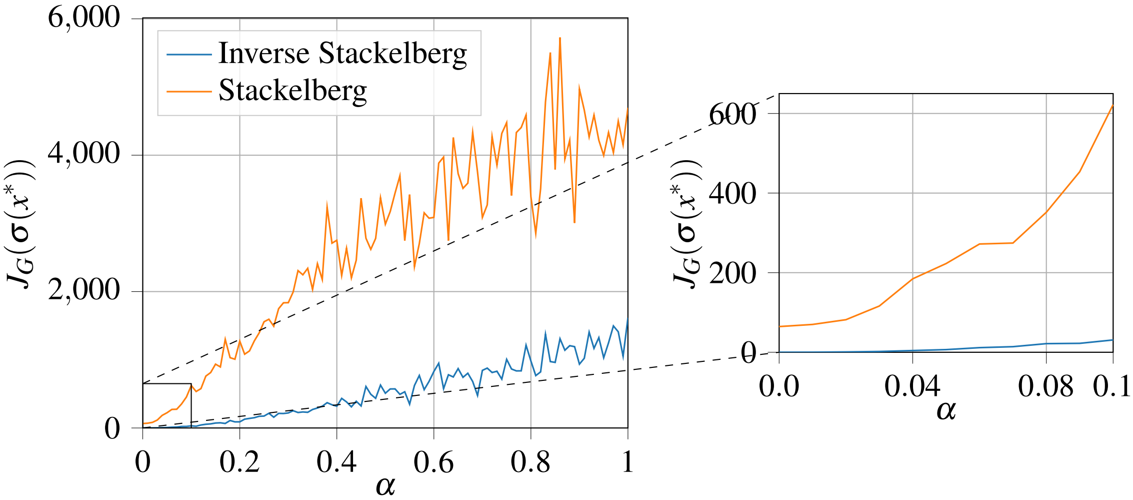

Figure 4: Robustness plot

4. CONCLUSIONS

In this paper we have analyzed a model for charge pricing of fleets of electric ride-hailing vehicles from the perspective of Stackelberg games as opposed to the pricing mechanism proposed in (Maljkovic et al., 2021). We proved that the Stackelberg game will also yield a unique Nash equilibrium of the game played between the ride-hailing companies, performed a grid search procedure to approximate the optimal price vector and compared the model to the one based on the Inverse Stackelberg game. The results from the new case study suggest that due to the simplicity of the pricing mechanism, the Stackelberg based model cannot match the performance of the Inverse Stackelberg based model. This becomes especially evident from the robustness analysis which in our opinion strongly motivates the usage of the pricing policies as opposed to the fixed pricing vector. With this in mind, in the future we plan to address the problem of accomplishing fair pricing among competing companies.

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REFERENCES

Basar, T., & Srikant, R. (2002). Revenue-maximizing pricing and capacity expansion in a many-users regime. In *Proceedings.twenty-first annual joint conference of the ieee computer and communications societies* (Vol. 1, p. 294-301 vol.1). doi: 10.1109/INFCOM.2002.1019271

Basilico, N., Coniglio, S., Gatti, N., & Marchesi, A. (2020). Bilevel programming methods for computing single-leader-multi-follower equilibria in normal-form and polymatrix games. *EURO Journal on Computational Optimization*, *8*. doi: 10.1007/s13675-019-00114-81

Brown, P. N., & Marden, J. R. (2020). Can taxes improve congestion on all networks? *IEEE Transactions on Control of Network Systems*, *7*(4), 1643-1653. doi: 10.1109/TCNS.2020.2992679

Coniglio, S., Gatti, N., & Marchesi, A. (2020). Computing a pessimistic stackelberg equilibrium with multiple followers: The mixed-pure case. *Algorithmica*, *82*. doi: 10.1007/s00453-019-00648-8

Geroliminis, N., & Daganzo, C. F. (2008). Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. *Transportation Research Part B: Methodological*, *42*(9), 759-770. doi: 10.1016/j.trb.2008.02.002

Groot, N., De Schutter, B., & Hellendoorn, H. (2012). Reverse Stackelberg games, Part I: Basic framework. *2012 IEEE International Conference on Control Applications*, 421-426. doi: 10.1109/CCA.2012.6402334

Internationa Energy Agency. (2021). Global EV outlook 2021. Retrieved from https://www.iea.org/reports/global-ev-outlook-2021

Ji, Y., Luo, J., & Geroliminis, N. (2014). Empirical observations of congestion propagation and dynamic partitioning with probe data for large-scale systems. *Transportation Research Record*, *2422*(1), 1-11. doi: 10.3141/2422-01

Laha, A., Yin, B., Cheng, Y., Cai, L. X., & Wang, Y. (2019). Game theory based charging solution for networked electric vehicles: A location-aware approach. *IEEE Transactions on Vehicular Technology*, *68*(7), 6352-6364. doi: 10.1109/TVT.2019.2916475

Ma, Z., Callaway, D. S., & Hiskens, I. A. (2013). Decentralized charging control of large populations of plug-in electric vehicles. *IEEE Transactions on Control Systems Technology*, *21*(1), 67-78. doi: 10.1109/TCST.2011.2174059

Maljkovic, M., Nilsson, G., & Geroliminis, N. (2021). *A pricing mechanism for balancing the charging of ride hailing electric vehicle fleets.* Retrieved from http://gustavnilsson.name/preprints/pricing.pdf (Sumbitted to 2022 European Control Conference (ECC).)

Paccagnan, D., Gentile, B., Parise, F., Kamgarpour, M., & Lygeros, J. (2016). Distributed computation of generalized Nash equilibria in quadratic aggregative games with affine coupling constraints. , 6123-6128. doi: 10.1109/CDC.2016.7799210

Paccagnan, D., Gentile, B., Parise, F., Kamgarpour, M., & Lygeros, J. (2019). Nash and Wardrop equilibria in aggregative games with coupling constraints. *IEEE Transactions on Automatic Control*, *64*(4), 1373-1388. doi: 10.1109/TAC.2018.2849946

Staňková, K., Olsder, G., & Bliemer, M. (2011). Bi-level optimal toll design problem solved by the inverse stackelberg games approach. *WIT Transactions on The Built Environment*, *89*, 871-880. doi: 10.2495/UT060841

Tushar, W., Saad, W., Poor, H. V., & Smith, D. B. (2012). Economics of electric vehicle charging: A game theoretic approach. *IEEE Transactions on Smart Grid*, *3*(4), 1767-1778. doi: 10.1109/TSG.2012.2211901

Wang, K., Xu, L., Perrault, A., Reiter, M. K., & Tambe, M. (2021). Coordinating followers to reach better equilibria:end-to-end gradient descent for stackelberg games. Retrieved from https://arxiv.org/abs/2106.03278

Zhang, J., Lu, J., Cao, J., Huang, W., Guo, J., & Wei, Y. (2018). Traffic congestion pricing via network congestion game approach. *Discrete and Continuous Dynamical Systems - S*, *14*. doi: 10.3934/dcdss.2020378

Zhang, L., Gong, K., & Xu, M. (2019). Congestion control in charging stations allocation with Q-learning. *Sustainability*, *11*(14). doi: 10.3390/su11143900