The effects of pricing and service configurations on a ride-pooling service with pick-up and drop-off points

Marko Maričić*[1], Rafał Kucharski[1], and Oded Cats[1]

[1]Department of Transport & Planning, Delft University of Technology, The Netherlands

SHORT SUMMARY

We investigate how the pricing of ride-pooling with pick-up and drop-off (PUDO) locations affects its level of service and operating costs when comparing to its competition such as private ride-hailing and door-to-door ride-pooling. We also examine how demand levels and the service settings affect these performance indicators. To this end, we extend an exact matching algorithm to the case of ride-pooling with PUDO and conduct experiments for the case of Amsterdam, the Netherlands. We find that total traveller utility can be further improved by 2.0% while reducing total vehicle hours by 2.2% when the discount offered for ride-pooling with PUDO is significantly larger than for door-to-door ride-pooling.

Keywords: ride-pooling, ride-hailing, ride-sharing, pick-up and drop-off points, walking.

Short summary: *104 words*
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1. Introduction

A key concept for future human mobility is ride-pooling where multiple travellers' trips are matched to a pooled ride, with respect to their routes and schedules. Accordingly, the provider can potentially reduce its operating costs due to increased vehicle occupancy, allowing for individuals to travel at a reduced fare (compared to private ride-hailing). Increasing the average occupancy could potentially result in the reduction of operator fleet size, alleviating traffic congestion, and reducing travel times for multiple types of road users. However, the extent of these benefits is still unclear and often disputed (Buhaug & Urdal, 2013; Ke, Yang, & Zheng, 2020; W. Li, Pu, Li, & Ban, 2019; Schaller, 2021).

The route and schedule flexibility of drivers is a constraint in ride-pooling as the matching of drivers and riders could be difficult at low demand levels. Cases with low density (e.g. off peak hours) could suffer from a state where the demand for trips attracts an insufficient supply and vice versa (Furuhata et al., 2013). Not only is a high matching rate critical for the success of ride-pooling services but also minimising the effort and inconvenience of travellers (Stiglic, Agatz, Savelsbergh, & Gradisar, 2015). Introducing pick-up and drop-off (PUDO) points to a ride-pooling service where travellers could walk to and from could promote higher matching rates and lower operating costs.

Meeting points are utilised in services such as *Uber Express Pool*. Here, multiple travellers in close proximity and with similar request times are matched to the same pooled ride. The travellers are then picked-up and/or dropped off at a certain distance from their respective origin and/or destination, to which they would have to walk.

Looking into existing research of ride-pooling with PUDO, Stiglic et al. (2015) propose an exact algorithm in which drivers and riders of a ride-pooling service are matched on a large scale. Space time windows can be used to formulate flexible pickup and delivery locations which is solved using Lagrangian relaxation (Zhao, Yin, An, Wang, & Feng, 2018) or through dynamic programming where a query of optimal meeting points are described as NP-hard (R.-H. Li, Qin, Yu, & Mao, 2015).

Fielbaum, Bai, and Alonso-Mora (2021) analyse ride-pooling with optimised PUDO walking locations using heuristics. Fielbaum (2021) shows that introducing PUDO points to vehicle routes in a ride-pooling system can potentially further reduce a third of vehicle costs. Goel, Kulik, and Ramamohanarao (2016) propose a ride-pooling scheme that chooses optimal pick-up locations from a set of fixed locations through maximising car occupancy and preserving user privacy. Gurumurthy and Kockelman (2020) uses an agent based simulator to explore the advantages of using PUDO locations for dynamic ride-pooling.

Flex-route transit systems share similarities with ride-pooling systems as users are able to request a PUDO point that is not located along the original bus route. Zheng, Li, Qiu, and Wei (2019) introduces meeting points that users would walk to, employing mixed integer programming to serve as many trip requests as possible while minimising the total trip time of the accepted travellers. Wang, Zeng, Ma, and Guo (2021) utilises discrete choice modelling and a vehicle routing model to generate routes and schedules of customised buses to potential travellers by proposing PUDO points.

Whether a flexible bus or a ride-pooling system, minimising the detour of a vehicle is a common objective; research has shown that the introduction of PUDO points decreases the costs of a system (reduction in vehicle hours) and increases the number of matched requests (increase in number of potential users). However, the extent of system-wide and ride-level benefits under various service configurations remain unknown. In essence, the configuration of PUDO points for a ride would depend on the operator and the travellers within that ride. For this, the operator would look to minimise its operating costs while travellers aim at maximising their utility. The latter consists of trade-off among factors such as: discount price, waiting time, or walking time to/from PUDO location.

We analyse the effects pricing configurations, service settings, and demand levels have on the operator performance and traveller level of service of a ride-pooling with PUDO points.

2. Methodology

This research builds upon an existing algorithm, ExMAS, which matches trips into attractive shared rides (Kucharski & Cats, 2020) by incorporating PUDO points into matched share rides where the resulting performance is then evaluated. We first briefly introduce the base algorithm, followed be the formulation of ride-pooling with PUDO.

Exact matching of attractive shared rides (ExMAS)

ExMAS uses efficient graph searches to adequately narrow the available trip search space and thereby determining the exact solution of attractive shared rides. Using utility based formulation, the potential modal split is obtained through comparing the attractiveness of a trip within a pooled ride to its hailed (private) counterpart.

Within ExMAS, travellers are offered a non-shared service-fare λ^{ns} (€/km) for a hailed ride. With respect to a traveller's origin and destination, the respective utility of a hailed ride (U_i^{ns}) can be computed. A traveller in a pooled ride will have the discomfort of sharing and a generally longer travel time where a different utility may be computed (U_i^s). Travellers are offered a shared service-fare λ^s for ride-pooling, for readability purposes the shared service fare can be expressed as relative discount λ where $\lambda = -(\lambda^s - \lambda^{ns})/\lambda^{ns}$. To increase increase the attractiveness of longer, less comfortable, pooled rides the shared service fare should be $\lambda^s < \lambda^{ns}$, or the discount for sharing should be $\lambda^{ns}(1-\lambda) < \lambda^{ns}$, as a traveller will find a shared ride attractive if and only if $U_{i,r}^s > U_i^{ns}$, or $U_{i,r} = U_{i,r}^s - U_i^{ns} > 0$. For travellers to select an offered pooled ride, all travellers in the respective ride must find it attractive (i.e. $\sum_{i \in \mathbf{Q_r}} U_{i,r} > 0$).

Ride-pooling with PUDO

As with hailed and door-to-door pooled rides, the utility of a traveller opting for ride-pooling with PUDO points, $U_{i,r}^{\tilde{s}}$, can be computed. Pooled rides with PUDO introduce a walking time, t_{w_i}, to travellers, resulting in longer total traveller travel times. To account for walking, a PUDO service fare, $\lambda^{ns}(1-\tilde{\lambda})$, is offered to travellers and should be $\tilde{\lambda} > \lambda$ where $U_{i,r}^{\tilde{s}} > U_{i,r}^s$ for attractiveness. Note that walking time, t_{w_i}, and in-vehicle travel time, $\tilde{t}_i$, differ for each PUDO configuration, thus $U_{i,r}^{\tilde{s}}$ varies with vehicle route.

As we also want to reduce operating costs, it is important to quantify the vehicle utility, $U_{r_v}^{\tilde{s}}$, a variable dependent solely on the total ride time, $\tilde{t}_{r_v}$, i.e. time between first picked-up and last dropped-off passenger. Note that the route of a pooled ride differs for each PUDO configuration, thus $U_{i,r}^{\tilde{s}}$ and $U_{r_v}^{\tilde{s}}$ vary for each feasible route. When summing the utilities of travellers in a ride, $U_r^{\tilde{s}} = \sum_{i \in \tilde{\mathbf{Q}}_r} U_{i,r}^{\tilde{s}}$, we can determine an optimum route with a given set of PUDO nodes. We maximise the utilities of the travellers within a ride and the respective vehicle using the following expression: $\text{argmax}\left(\alpha U_{r_v}^{\tilde{s}} + (1-\alpha) U_r^{\tilde{s}}\right)$.

For a given ride, travellers would want to minimise their walking time while the operator would want to minimise the detour induced by pooling. We can use a service setting, α, to dictate the perspective for which we configure the route for. On a trip level we can seek for a route for either the traveller ($\alpha \approx 0$), the vehicle ($\alpha \approx 1$), or treating the travellers and vehicle equally ($\alpha = 0.5$). An example of the computable routes can be seen in Figure 1.

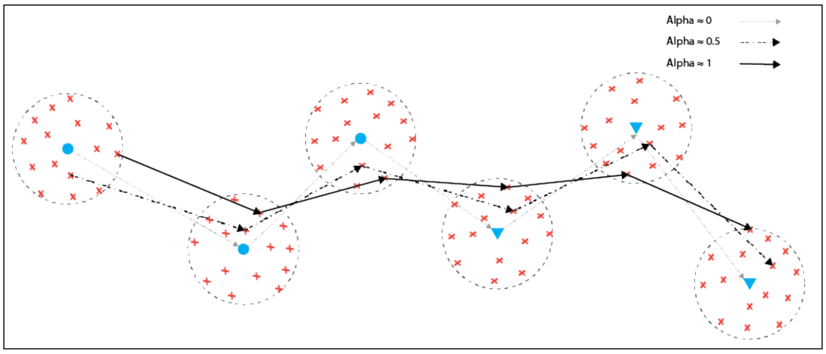


Figure 1: Visualisation of the ride-sharing with PUDO locations problem. Traveller origins and destinations are visualised with points and triangles, respectively. PUDO locations are visualised with crosses which are limited to the dotted circle bounded by a walking distance radius. The different routes are obtained by setting a service parameter α in the objective function.

The algorithm utilises matched and selected pooled rides that ExMAS produces and configures the resulting routes by allocating PUDO nodes to each traveller in a pooled ride. Per ride, the algorithm conducts an exhaustive route search with heuristics on the possible routes with respect to the amount of PUDO nodes. The code is publicly availabe on a *GitHub* repository (Maricic, 2021).

The algorithm essentially utilises ExMAS generated ride data and further configures already matched and selected door-to-door pooled rides. Each trip request is allocated a set of PUDO nodes (respective to the origin and destination) which is bounded by a predefined walking radius. Due to the number of PUDO nodes that can be assigned to the respective travellers, there could be up to 10^7 possible routes for a given ride.

For a given ride, each feasible route with PUDO points is iterated through where a route is constantly compared to a candidate route which is the best route that was determined up until that iteration. The shortest route approach is utilised except for the last trip pair where utility of the travellers in the ride is also considered. The total utility ($\alpha U_{r_v}^{\tilde{s}} + (1-\alpha) U_r^{\tilde{s}}$) is then computed and then compared to the candidate route, where we seek to maximise the utility.

Undoubtedly, an exhaustive search is impractical and therefore we use heuristics to decrease the size of the search space. Once the search space has been explored, the best candidate route along with relevant information is saved.

3. Application

Case study

The algorithm is applied to an Amsterdam network graph, generated with OSMnx (Boeing, 2017) and activity based synthetic demand using Albatross (Arentze, 2005). ExMAS (Kucharski & Cats, 2020) is used to assign a series to generate a set of attractive shared rides for which all input request then be used for PUDO route search.

Experimental set-up

The experiment set-up is summarised in Table 1. The variation of door-to-door sharing λ and PUDO sharing $\tilde{\lambda}$ discounts is intended to study the effects of pricing configurations on ride-pooling service with PUDO while the variables α and Q are used to investigate the effects of service settings and demand levels, respectively.

Table 1: Experimental design

Parameter	Symbol	Range of values	Unit
Door-to-door sharing discount	λ	{0.15, 0.2, 0.25, 0.3, 0.35}	-
PUDO sharing discount	$\tilde{\lambda}$	{0.2, 0.3, 0.4, 0.5}	-
Service setting	α	{0.5, 0.75, 0.95}	-
Demand level (No. of trips)	Q	{1000, 2000, 3000, 4000}	[trips/hr]

Quantifying the results of the experiments shown in Table 1 is conducted using a series of performance indicators. The relative-difference in traveller utility between PUDO ride-sharing and door-to-door, $u_{i,r}^s$, ride-sharing is shown in Equation 1.

$$\Delta u_{i,r}^s = (U_{i,r}^{\tilde{s}} - U_{i,r}^s)/U_{i,r}^s \tag{1}$$

When summing $u_{i,r}^s$, the utilities of all travellers in a pooled ride with PUDO can be compared to the door-to-door counterpart, as seen in Equation 2.

$$\Delta U_r^s = \sum_{i \in \mathbf{Q_r}} \Delta u_{i,r}^s \tag{2}$$

Note that, a traveller would deem a pooled ride with PUDO attractive if and only if $\Delta u_{i,r}^s > 0$ where all travellers must find a ride attractive for it to be selected, thus $\Delta U_r^s > 0$ for selecting a pooled ride with PUDP.

Similarly, in-vehicle traveller travel times between PUDO, $\tilde{t}_{i,r}$, and door-to-door ride-pooling, $\hat{t}_{i,r}$, are compared using Equation 3.

$$\Delta t_{i,r}^s = (\tilde{t}_{i,r} - \hat{t}_{i,r})/\hat{t}_{i,r} \tag{3}$$

We also compare ride travel time of PUDO and door-to-door ride-pooling using Equation 4.

$$\Delta T_r^s = (\tilde{t}_r - \hat{t}_r)/\hat{t}_r \tag{4}$$

Furthermore, system-wide indicators are also used to quantify the benefits (and drawbacks) of a tri-modal system, a system PUDO and door-to-door pooled rides and private rides operate (indicated with $\tilde{s}$), where compared to a dual-modal system, a system of private hailed and door-to-door pooled rides (indicated with s), or when compared a single modal system, a system of only private hailed rides of only private hailed rides (indicated with ns) (Kucharski & Cats, 2020). These are as follows:

$$U = \sum_{i \in \mathbf{Q}} U_{i,r} \tag{5}$$

$$T_v = \sum_{i \in \mathbf{R}} t_r \tag{6}$$

$$T_q = \sum_{i \in \mathbf{Q}} t_{i,r} \tag{7}$$

$$I = \sum_{i \in Q} \lambda_i \cdot l_i \tag{8}$$

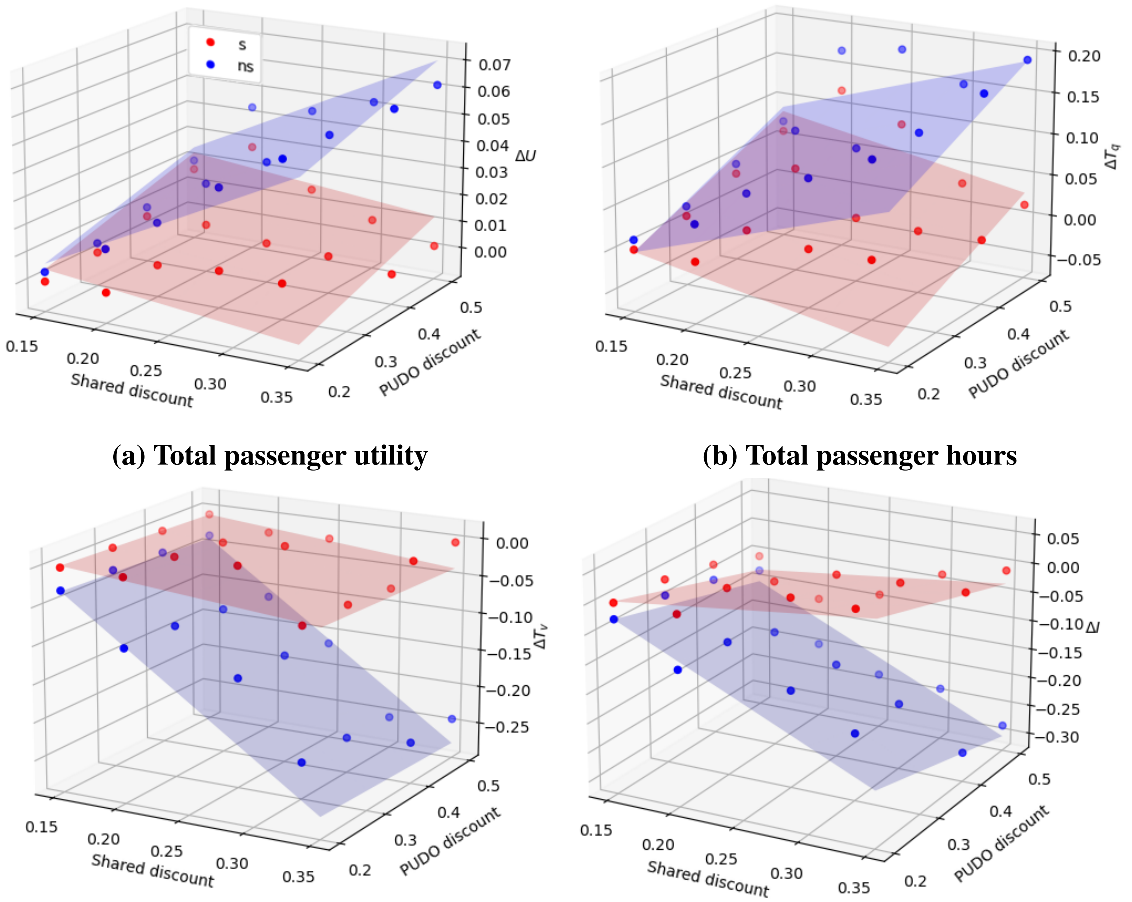
$$\Delta T_q^s = (T_q^{\tilde{s}} - T_q^s)/T_q^s \tag{9}$$

The total traveller utility, vehicle-hours, passenger-hours, and revenue of the system are computed using Equations 5, 6, 7, and 8, respectively. To compare system-wide indicators between the three different systems, we compute relative differences of the aforementioned system-wide indicators. An example can be seen in Equation 9 where we compare the total passenger hours of the tri-modal system to the dual-modal.

4. Results

Ride-pooling with PUDO under various pricing strategies

Figure 2 depicts a series of system-wide indicators when the system is set to various λ and $\tilde{\lambda}$. Figure 2a shows that total passenger utility generally improves for increasing λ and $\tilde{\lambda}$. For larger λ, ΔU^{ns} is largest while ΔU^{ns} is lowest. This indicates that ΔU^{ns} improvement at high λ is largely due to the door-to-door pooled rides rather than pooled rides with PUDO. The ΔU^{s} is largest (around 2%) when the difference between λ and $\tilde{\lambda}$ is largest. At these points, ΔU^{s} and ΔU^{ns} are quite similar to each other due to the low market share of door-to-door pooled rides.



(a) Total passenger utility **(b) Total passenger hours**

(c) Total vehicle hours **(d) Total revenue**

Figure 2: Correlations of relative differences of an array of system-wide performance indicators with λ and $\tilde{\lambda}$. In this case, PUDO ride-pooling is compared to the door-to-door and private alternatives, visualised in red and blue respectively. $Q = 1000$ trips and $\alpha = 0.75$ for all experiments here.

Figure 2b shows that the trends seen in Figure 2a are almost identical to those seen with the utility improvement. For all discounts, passenger hours increase (regardless if compared to non-shared or shared rides) as walking times are incorporated in the passenger travel time.

Figure 2c shows that ΔT_v^s has close to no correlation with the discount prices offered when comparing to ΔT_v^{ns}. Notwithstanding, ΔT_v^s can be up to 2.2% when differences between λ and $\tilde{\lambda}$ are large. An almost linear negative trend is seen with ΔT_v^{ns} and λ where up to 25% total vehicle hour reductions occur when $\lambda = 0.35$. At this λ, ΔT_v^{ns} is dictated by the large number of selected door-to-door pooled rides.

Figure 2d shows that any discount configuration produces a decrease in revenue when compared to a system of only private rides. ΔI^{ns} can be down by 28.1% when both λ and $\tilde{\lambda}$ are at their largest. At the aforementioned discounts, ΔI^{s} is smallest due to the much larger number of door-to-door than PUDO pooled rides. When the difference between discounts is largest, ΔI^{s} is -5.5%. However, when $\lambda = 0.15$ the difference between ΔI^{s} and ΔI^{ns} is fairly small due the small number of attractive pooled rides determined by ExMAS

Ride-pooling with PUDO under various service settings

It was found that most ride-pooling travellers opt for ride-pooling with PUDO when $\alpha = 0.5$ while a larger share opts for door-to-door pooling when $\alpha > 0.75$. The share of private-hailed rides in our experiment does not change since we only configure selected door-to-door pooled rides.

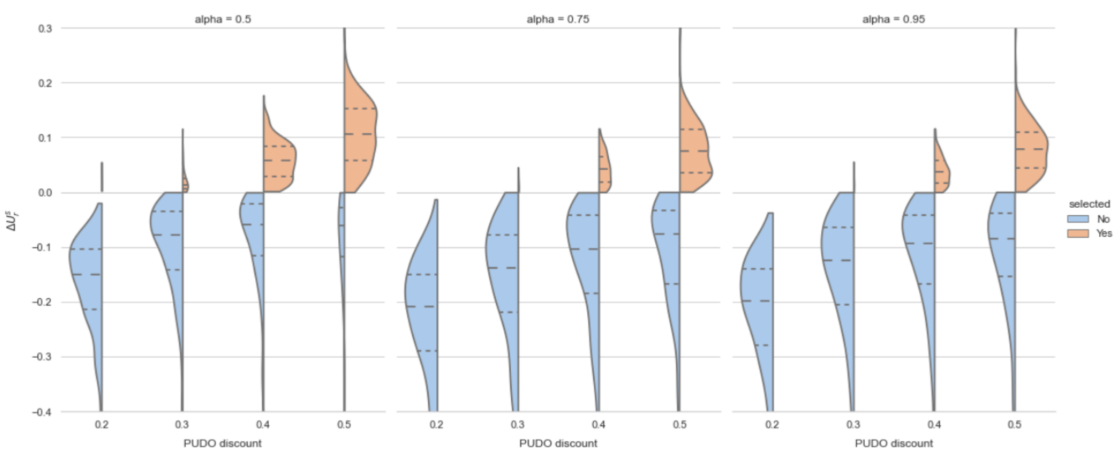
The effect of α on ΔU^{s} is minimal when the difference between λ and $\tilde{\lambda}$ is small, likely due to the small number of attractive pooled rides. when the difference between λ and $\tilde{\lambda}$ increases, the largest improvements in utility occur when $\alpha = 0.5$. The differences in performance when $\alpha > 0.75$ was found to be insignificant. Similar patterns are seen when examining ΔT^{s}_{q} for various α values.

Next, we find that total vehicle hours are reduced the least when $\alpha = 0.5$. The largest vehicle hour reduction relative to a system of private and door-to-door pooled rides, was around 2.5% when $\lambda = 0.2$, $\tilde{\lambda} = 0.5$, $\alpha = 0.95$.

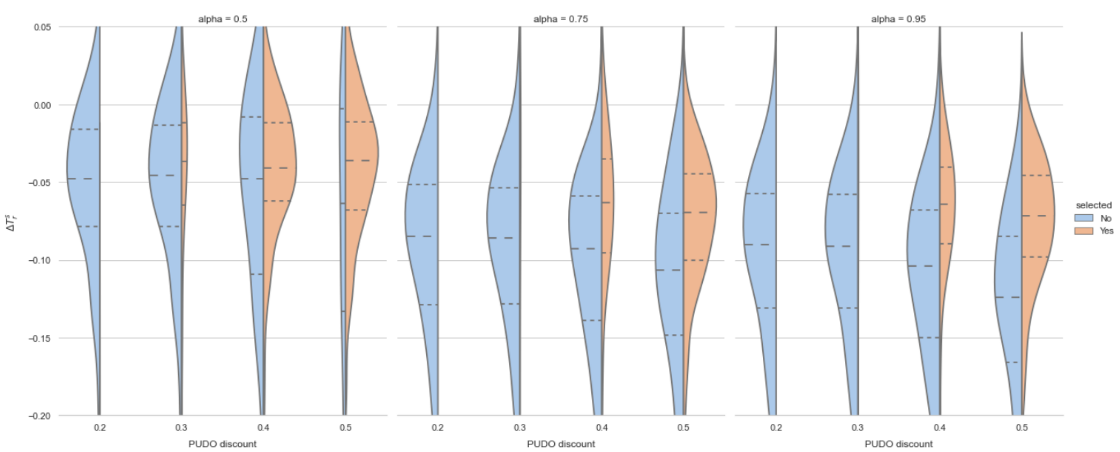
To dive deeper, Figure 3 visualises the distributions of ride-level utility improvements and vehicle travel time reductions. Figure 3a shows that mean ΔU^{s}_{r} is largest for both selected and non-selected PUDO pooled rides when $\alpha = 0.5$. However, it seen in Figure 3b that $\Delta T^{s}_{v_r}$ is lowest when $\alpha = 0.5$ where a minor difference in performance exists between selected and non-selected rides. Interestingly, when $\alpha \geq 0.75$, selected PUDO rides have a significantly lower mean $\Delta T^{s}_{v_r}$ than non-selected PUDO rides. Reasoning for this can be seen in Figure 4 where $\alpha = 0.5$ induces much shorter walking times. Again, the difference between selected and non-selected rides are much larger when $\alpha \geq 0.75$ which shows that $\Delta T^{s}_{v_r}$ is linked closely to the t_{w_i}. Furthermore, larger values of $\tilde{\lambda}$ only slightly increases t_{w_i}, indicating that larger discounts do not persuade travellers to walk farther but rather persuade travellers to select rides with larger t_{w_i}.

Examples of routes generated through different service settings are shown in Figure 5. Here, the route of a pooled ride with three travellers is further configured with the use of PUDO nodes. When $\alpha = 0.50$, the travellers are subject to lower walking times than when $\alpha = 0.95$.

We also find that traveller utility increases linearly with $\tilde{\lambda}$ offered and decreases with increasing α. Larger demand levels facilitate the matching of higher degree pooled rides which help with further reducing total vehicle hours. Incorporating PUDO locations help further reducing the travel times of vehicles and even more so with increasing Q. Increasing Q correlates negatively with ΔU^{s}_{r}, likely due to the larger number of higher degree rides. The α setting could be used as a mediator for improving either the ΔU^{s}_{r} or ΔT^{s}_{r} depending on the requirements for the time of operation.



(a) Ride level utility improvement ΔU_r^s



(b) Ride Vehicle hour reduction $\Delta T_{v_r}^s$

Figure 3: With $Q = 2000$ trips and $\lambda = 0.2$, distributions of ride-level (with respect to shared rides) indicators for PUDO optimised pooled rides for various $\tilde{\lambda}$ and α. Distributions are split into selected and non-selected PUDO pooled rides categories and shaded with orange and blue, respectively.

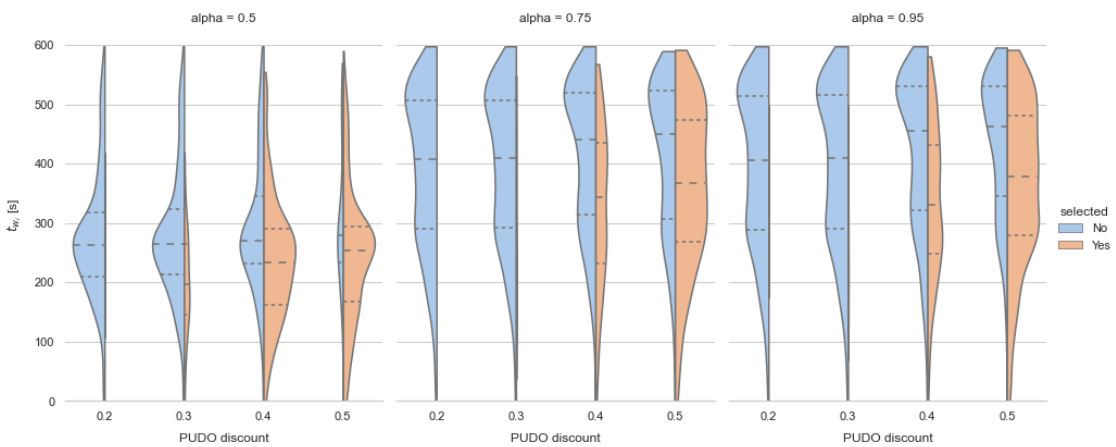


Figure 4: With demand $Q = 2000$ trips and $\lambda = 0.2$, distributions of walking times for travellers are split into selected and non-selected PUDO pooled rides, coloured with orange and blue respectively.

(a) $\alpha = 0.50$ (b) $\alpha = 0.95$

Figure 5: The route of a door-to-door pooled ride with three travellers, visualised in blue, and the new route with PUDO points in green, when using different service settings α.

5. Conclusion, limitations, and recommendations

We investigated the effect pricing configurations, service settings, and demand levels have on a ride-pooling service with pick-up and drop-off points. This was done by first building upon an existing and scalable algorithm. The existing utility information was extend and incorporated into a route search algorithm that is able to assess the potential of a ride's PUDO point configuration with respect to the vehicle and travellers' utility. We demonstrate the, the algorithm was able to determine new routes for already matched pooled rides by introducing PUDO locations that travellers walk to and from.

The total utility of travellers generally improved with larger PUDO discounts offered while the revenue due to more travellers opting for pooled rides with PUDO. However we found that ride-pooling with PUDO points performs best when the difference between the two offered discounts is large. To compensate travellers, the PUDO discount should be around twice as large as the sharing discount. We were able to find that at this discount configuration, ride-pooling with PUDO was able to reduce a further 1.0% of total vehicle hours while total revenue decreased a further 3.7%. For some operators, this could be a risky operation due the insufficient reduction in operating costs.

In this research, we also examined how the performance of ride-pooling with PUDO is affected by service settings and demand level. Generally, in a ride-pooling network, larger demand levels increase the likelihood of matching more travellers within a single pooled ride, thus allowing for further savings in vehicle travel time. For an operator to introduce a ride-pooling system with PUDO to a city, minimal critical mass of demand level is required for the operator to make sufficient revenue. Given a certain demand level, the service setting could be used as a mediator for either improving the traveller utility or the vehicle traveller time, depending on the requirements for the time of operation. Alternatively, with the service setting an operator could control the the attractiveness of ride-pooling with PUDO. This could help with fleet management when supply exceeds demand and vice versa.

As part of future research the findings discussed should be verified with similar algorithms that tackle the ride-pooling with PUDO problem, such as the ones presented by (Fielbaum, 2021) and (Fielbaum et al., 2021). Furthermore, this research takes on a planning approach rather than a real-time optimisation where no traffic (congestion) is included. The geographical position of PUDO nodes would differ if traffic were to be included where consideration must be made to minimise traffic disruption when picking travellers up.

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