Performance of Synthetic Road Networks with Dedicated Streets for Altruists

Alonso Espinosa Mireles de Villafranca*[1] and Claudio Roncoli[2]

[1]Postdoctoral Reasercher, Department of the Built Environment, Aalto University, Finland

[2]Assistant Professor, Department of the Built Environment, Aalto University, Finland

SHORT SUMMARY

We present an empirical study of network ensembles under static traffic assignment with two vehicle classes: an altruistic vehicle class (AVs) – potentially automated vehicles that are optimally routed – and a selfishly routing class, corresponding to human-driven vehicles (HVs). We assess a management measure that consists on reserving some network links for exclusive use by AVs, to mitigate the detrimental effects of HVs on the per-vehicle cost of AVs. We formulate a capacitated, multi-class, static traffic assignment problem (STAP). We solve the STAP as a single-objective, convex optimisation problem with additional flow constraints on AV-exclusive links. We find that randomly selecting the AV-exclusive links can be detrimental at low penetration rates. However, having several sets of links to choose from and selecting the best performing link-set for the traffic conditions steers the system closer to optimal even for modest penetration rates (e.g., 30%).

Keywords: Altruistic Routing, Automated Vehicles, Mixed Equilibrium, Network Design, Network Segregation, Static Traffic Assignment

1. INTRODUCTION

Connected and automated vehicles are being presented as one of the main components of future of mobility systems; however, market projections of their introduction to the vehicle fleet suggest decades of coexistence with human-driven vehicles (Department for Transport, 2021; Lavasani, Jin, & Du, 2016) . In addition, depending on the new vehicle ownership models that may develop, as well as the new policies that may be deployed, the impact of automation on urban traffic remains highly uncertain. Furthermore, in the future, communication technologies, such as vehicle-to-infrastructure communication, may enable a much more flexible management of the existing infrastructure, where novel traffic management strategies may be devised and implemented; for example, a traffic management authority may dynamically allocate exclusive right-of-way to different types of vehicles (whether automated or not) along specific streets in the network.

In this paper, we investigate a mixed traffic scenario in which connected and automated vehicles are part of a single fleet that is centrally managed by a transport authority that routes all journeys with the goal of optimising the average travel time of all vehicles on the network. That is, a proportion of the vehicle fleet is composed of *altruistic vehicles* (AVs), whereas the remaining human-driven vehicles (HVs) behave as *selfish vehicles*. This is modelled as a *static traffic assignment problem* (STAP). The AVs attempt to achieve *system optimum* (SO) by routing themselves in a way that minimises the average travel time of the whole network. Meanwhile, the HVs follow *user equilibrium* (UE) In combination with this, we consider that a subset of streets on the network can be allocated exclusively to AVs to make altruistic travel more attractive.

The*mixed equilibrium* (ME) system is modelled as a multi-class *static traffic assignment problem* (STAP) (Dafermos, 1972) following (Van Vuren, Van Vliet, & Smith, 1990). Van Vuren et al. studied this in the context of route guidance systems. They show that while total costs are reduced, SO vehicles experience higher per-vehicle costs. Thus one can expect that altruistic routes would not be chosen without further incentives.

The more recent results of (Yang, Zhang, & Meng, 2007) confirm that the mixed equilibrium (ME) can indeed improve significantly with respect to UE. However, it is possible to introduce system-optimising vehicles without any cost improvements to the system until a threshold penetration rate is reached. Additionally, the SO vehicle class experiences an uneven distribution of route costs, that makes the higher cost SO routes highly unattractive in comparison to UE costs.

In the 80s and 90s, the possibility of nudging the system closer to SO by giving drivers routing options that lower system costs became apparent (Watling & van Vuren, 1993). Studies have covered different guidance strategies, as well as modelled user classes in different ways to incorporate different aspects of stylised realistic behaviour, for example, by using *stochastic assignment* to capture both perception error of non-guided users, as well as possible estimation errors by guidance systems (van Vuren & Watling, 1991). How and at which stage of the journey the information is presented has also been studied, for example in (Mahmassani & Chen, 1991), results show that the interaction between user behaviour and how suggestions are delivered is a crucial factor in whether any performance improvements materialise.

Navigation apps, like Waze and Google Maps, are now ubiquitous yet their impact on traffic flows is still being understood. In (Bianchin & Pasqualetti, 2020), the authors show how route guidance can result in oscillatory congestion patterns. Furthermore, it has been suggested that the effects of users accepting fastest-route suggestions can lead increases in travel times as adoption increases (Festa & Goatin, 2019).

A key feature of the current use of navigation apps is that they are used in conjunction with selfish routing. In future mobility systems, connected and automated vehicles (CAVs) can provide a means of introducing altruistic routing, if, for example, a centrally managed fleet of AVs is deployed, whose objective is to minimise total system costs. If the cost of taking an AV can be kept below, or comparable to that of HVs, then a suitable penetration rate can arise naturally out of selfish mode choice. This can also be facilitated by the advent of CAVs (e.g., at SAE levels 4 or 5) and vehicle-to-infrastructure communication. Special dedicated links for CAVs have been suggested as a way of exploiting the benefits of CAVs, for example, in conjunction with platooning and headway reduction (see (Shladover, Su, & Lu, 2012; Spiliopoulou et al., 2017; Papamichail et al., 2019)), whilst avoiding the mixture of CAVs and HVs that might disrupt the efficiency gains.

Premise of this paper

Our aim, in contrast to papers that focus on solving the STAP or the network design problem, is to understand the implications of altruistic routing *in general* across an ensembles of networks with varying topologies, coupled with an infrastructure management scheme – allocating links in the networks for exclusive AV use – to mitigate the costs absorbed by the vehicles that deviate from UE routing.

In this sense, our objective is twofold. First, to encompass networks with different types of structures. Second, to compare how a 'reasonable', yet ad hoc, way of choosing AV exclusive lanes – that might mimic how transport authorities face real-world constraints – compares in effectiveness with designating the AV exclusive lanes via a rigorous optimisation approach.

We present results for ensembles of many random-generated networks that span different types of morphologies. Results are given for a range of demand values and for different penetration rates of AVs. To account for the dynamical behaviour that typically characterises a traffic network, we test several subsets of AV-exclusive links for each network, selecting the flows and costs of the best performing subset. Thus, for each demand level / penetration rate pair, the best performing subset is chosen; this aims at mimicking that given knowledge of the traffic state of the network (demand, class composition), the infrastructure can adapt in an optimised way.

2. METHODOLOGY

Our methodology is composed of the following three stages. The first consists of the generation of the synthetic networks that are used in the numerical experiments, as well as selecting parameters for their cost functions and a suitable OD pair.

The second stage consists of selecting the links of the networks that will be reserved for AV use. In this step, to guarantee some measure of realism even in a highly stylised management method, we emphasise the need to ensure that all possible OD pairs (i.e., all pairs of nodes) in a network must remain connected for both classes. Additional links are selected by sampling paths between the OD pair and incorporating links used by these paths into the set that guarantees connectedness.

The third stage covers the calculation of the traffic assignment. We express the Wardrop equilibrium (Wardrop, 1952) of both vehicle classes (AVs and HVs) as the solution of an optimisation problem with a single objective (following (Van Vuren et al., 1990)).

The complete experimental workflow, from network generation to obtaining ensemble results for the link-segregated ME is represented in Figure 1. The input parameters are those for the network generation; the path sampling parameters used in the path-sampling algorithm; and the range of penetration rate values.

Network Model and Demand Structure

We use the $\alpha\beta$-network model developed in (Espinosa Mireles de Villafranca, Connors, & Wilson, 2017; Espinosa Mireles de Villafranca, 2020) to generate synthetic street networks that are directed random planar graphs. These

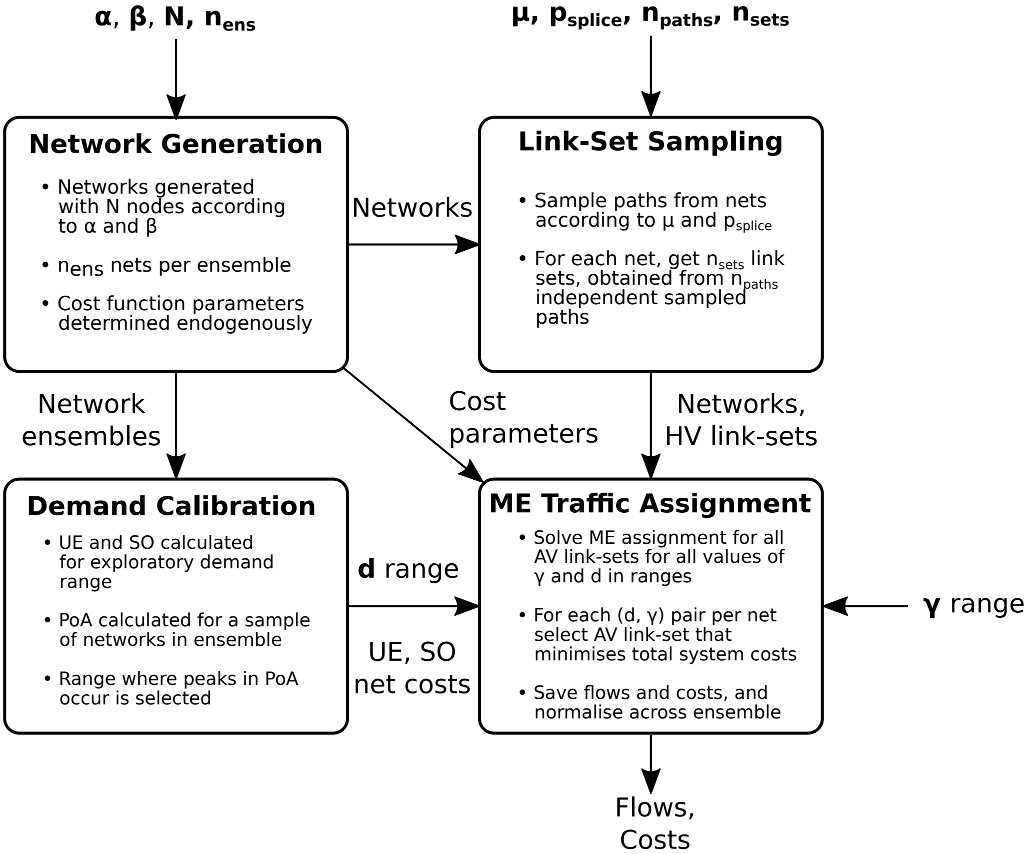


Figure 1: Components of the proposed methodology, with input parameters and relevant outputs from each step. See remaining of section 2 for details on notation and parameter definitions

networks resemble realistic urban networks to some extent. The model generates networks according to two structural parameters α and β that control for the randomness of node distribution and the density of edges, respectively. Figure 2 shows some network instances for different values of α.

The node-placement parameter α changes the distribution of the nodes between a perfect lattice ($\alpha = 0$) and a uniformly random distribution ($\alpha = 1$) inside the unit square ($[0,1]^2$). Therefore, the model can capture different morphologies from very grid-like to networks that are more random. The edges of the network are determined by constructing their β-skeleton (Jaromczyk & Toussaint, 1992), here β controls for the density of edges. The β-skeleton has been used to reconstruct urban street networks; e.g., in (Osaragi & Hiraga, 2014), with high rates of agreement.

For each edge i in the network we use flow-dependent affine cost functions, c_i of the form

$$c_i(f_i) = a_i + b_i f_i, \tag{1}$$

where f_i is the vehicular flow on the edge, while a_i and b_i are edge-specific parameters that represent the free flow travel time and the sensitivity to congestion, respectively.

The a_i, are set proportional to the (euclidean) length of the edge, while the b_i are allocated with a heuristic that ensures that networks with different amounts of edges are comparable: all intersections are allocated the same node capacity, which is then evenly split amongst the incoming links at each node.

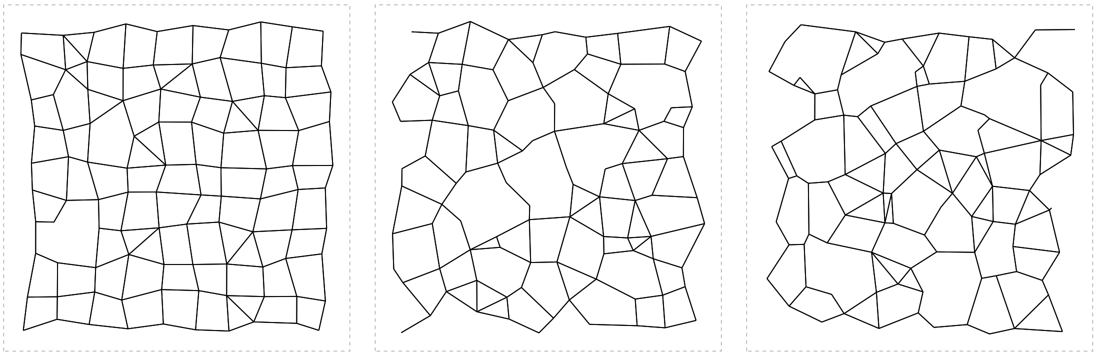
This heuristic ensures that all networks have the same supply of infrastructure, which can be set to unity so that

$$\sum_{i \in \mathscr{E}} \frac{a_i}{b_i} = 1, \tag{2}$$

where m is the number of edges in the network. Additionally, we require that all nodes in a network have the same "node capacity" which is allocated evenly among each node's incoming edges,

$$b_i = k_i \sum_{v \in \mathscr{V}} \frac{1}{k_v} \sum_{j \in \mathscr{I}_v} a_j, \tag{3}$$

where $\mathscr{I}_v$ is the set of edges that are incoming at node v and k_v is the in-degree of v. Note that, k_i is the in-degree of the node to which edge i is incident. A more detailed description can be found in (Espinosa Mireles de Villafranca et al., 2017; Espinosa Mireles de Villafranca, 2020).



(a) (b) (c)

Figure 2: Some examples of $\alpha\beta$-networks composed of 100 nodes each. They are all generated with $\beta = 1.5$ with (a) $\hat{\alpha} = 0.5$, (b) $\hat{\alpha} = 0.5$, and (c) $\hat{\alpha} = 0.5$

Given affine cost functions, the quotient a_i/b_i can be naively interpreted as a type of road area (if units are ignored), where $1/b_i$ should scale with the number of lanes, implying that wider roads are less sensitive to congestion. Therefore, equation 2 can be loosely interpreted as assigning a constant road surface area for all networks in an ensemble, ensuring a constant infrastructure supply.

For simplicity, we use a single OD pair. The origin and destination are chosen to be far apart form each other so that the vehicles can make use of as much of the network as possible. As the networks are constructed inside the unit square, the origin is selected as the node closest to the bottom left corner, point $(0,0)$, and the destination as the node closest to the top right corner, point $(1,1)$.

Selection of AV Exclusive Links

To constrain the complexity of the problem, as well as to ensure that all pairs of nodes remain connected for both vehicle classes, not all links are candidates to be AV exclusive links. Since we are restricting the network for the HVs, we will first determine the links that *will* be usable by them, and obtain the AV exclusive links as the complementary set of links.

First, for each network a *minimum spanning tree* (MST) is found (e.g., via Kruskal's algorithm (Kruskal, 1956)), and added to the link set that allows HVs. This guarantees connectivity between any pair of nodes. Then, this set is augmented by sampling paths between origin and destination using the the Metropolis-Hastings (MH) method proposed in (Flötteröd & Bierlaire, 2013). The inverse temperature parameter, that controls for how close to shortest-paths the sampled ones are, is set to $\mu = 3$.

Figure 3 shows a network with paths sampled using the MH algorithm, as well as the complete link-set to which the HVs are restricted once the links belonging to the MST between the origin and destination have been added to the HV link-set.

Traffic Assignment

We express the ME STAP as a single-objective convex optimisation problem following (Van Vuren et al., 1990) and (van Vliet, Bergman, & Scheltes, 1986) and combine the different experienced costs for each class in a single Beckmann-like objective functional.

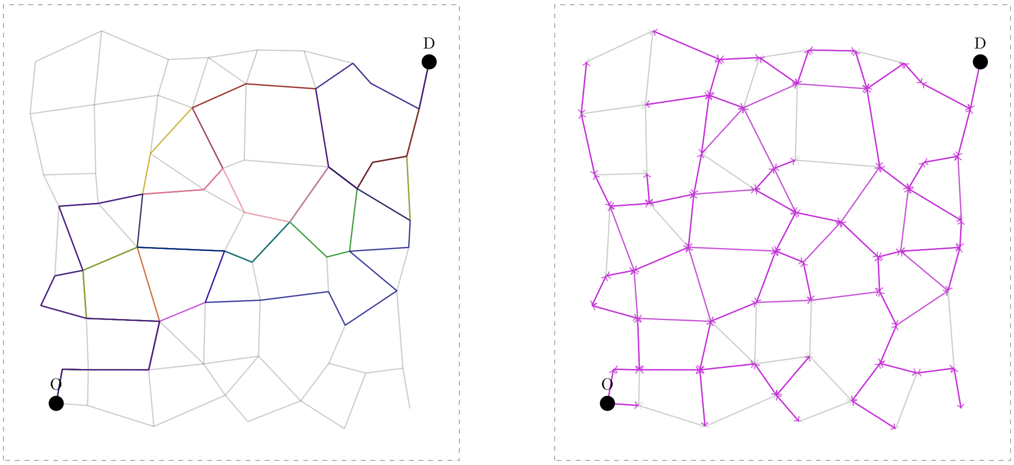
The mixed equilibrium assignment can be obtained by minimising the following objective function

$$T(\mathbf{f}^{\mathrm{HV}}, \mathbf{f}^{\mathrm{AV}}) = \sum_{i\in\mathscr{E}} \int_0^{f_i^{\mathrm{AV}}+f_i^{\mathrm{HV}}} b_i s\, ds + \sum_{i\in\mathscr{E}} a_i f_i^{\mathrm{HV}} + \sum_{i\in\mathscr{E}} \frac{a_i}{2} f_i^{\mathrm{AV}}, \tag{4}$$

where $\mathbf{f}^{\mathrm{HV}}$ is the flow vector of HVs, $\mathbf{f}^{\mathrm{AV}}$ the one for AVs with a_i and b_i the cost function parameters for the links. The first term captures the flow-dependent term of the costs, while the second and third terms capture the (transformed) free flow costs for the HVs and AVs, respectively.

We solve the problem in the link-node formulation (Patriksson, 2015), where the flow conservation constraints are imposed at each node for each vehicles class (AVs and HVs). That is

$$\begin{aligned} A\,\mathbf{f}_{pq}^{\mathrm{HV}} &= \mathbf{d}_{pq}^{\mathrm{HV}}, \qquad \forall (p,q) \in \mathscr{C}^{\mathrm{AV}} \\ A\,\mathbf{f}_{pq}^{\mathrm{AV}} &= \mathbf{d}_{pq}^{\mathrm{AV}}, \qquad \forall (p,q) \in \mathscr{C}^{\mathrm{AV}}, \end{aligned} \tag{5}$$



(a) **(b)**

Figure 3: (a) Paths sampled with Metropolis-Hastings algorithm (8 paths, with $\mu = 3$). (b) Edges that belong to paths or the MST; note that HVs are allowed in the majority of the network

where A is the link-node incidence matrix of the road network and $\mathbf{d}_{pq}^{\text{class}}$ is a vector that indicates the sink and source demands for each OD pair, defined as

$$(\mathbf{d}_{pq}^{\text{class}})_i = \begin{cases} -d_{pq}, & \text{if } i = p \\ d_{pq}, & \text{if } i = q \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Additionally, we impose flow constraints for the HVs on the AV-exclusive links, $\mathscr{A}^{\text{AV}}$,

$$f_i^{\text{HV}} = 0, \quad \text{for } i \in \mathscr{A}^{\text{AV}} \tag{7}$$

In summary, the assignment is found by solving the minimisation problem,

$$\begin{aligned} \min_{\mathbf{f}^{\text{HV}},\, \mathbf{f}^{\text{HV}}} \quad & T_k(\mathbf{f}^{\text{HV}}, \mathbf{f}^{\text{HV}}) \\ \text{s.t.} \quad & \mathbf{f}^{\text{HV}} \geq \mathbf{0} \\ & \mathbf{f}^{\text{HV}} \geq \mathbf{0} \\ & \\ & A\,\mathbf{f}_{pq}^{\text{HV}} = \mathbf{d}_{pq}^{\text{HV}}, \quad \forall (p,q) \in \mathscr{C}^{\text{HV}} \\ & A\,\mathbf{f}_{pq}^{\text{AV}} = \mathbf{d}_{pq}^{\text{AV}}, \quad \forall (p,q) \in \mathscr{C}^{\text{AV}} \\ & \\ & f_i^{\text{HV}} = \sum_{k \in \mathscr{C}^{\text{HV}}} f_{ki}^{\text{HV}}, \quad \forall i \in \mathscr{A} \\ & f_i^{\text{AV}} = \sum_{k \in \mathscr{C}^{\text{AV}}} f_{ki}^{\text{AV}}, \quad \forall i \in \mathscr{A} \\ & \\ & f_i^{\text{HV}} = 0, \quad \forall i \in \mathscr{A}^{\text{AV}}. \end{aligned} \tag{8}$$

Problem 8 is solved for each network in the randomly generated ensemble and the costs are averaged across all networks. The demand for each class is prescribed by the experimental demand range together with the penetration rate γ of AVs. Where, $d^{\text{HV}} = (1-\gamma)d$ and $d^{\text{AV}} = \gamma d$, where d is the total demand.

Since our networks are synthetic, the demand range is chosen according to the *price of anarchy* (PoA), which is defined as the ratio of the costs between the UE and SO assignments (Roughgarden, 2006; Correa, Schulz, & Stier-Moses, 2008). We select the demand range so that it contains the region with the highest PoA values. This guarantees that the traffic conditions on the networks are such that there are indeed gains to be made (in terms of system costs) by pushing the system towards SO.

For each network five different AV-exclusive link sets are chosen and for each demand-penetration rate pair the best performing one is chosen. The parameters for our numerical experiments are shown in table 1.

Table 1: Parameters used in numerical experiments. N: number of nodes in each network, $\hat{\alpha}$: griddedness parameter, β: network wiring parameter, $n_{\mathbf{ens}}$: networks per ensemble, μ, $p_{\mathbf{splice}}$: MH sampling parameters, $n_{\mathbf{paths}}$: number of paths sampled for HV link selection, $n_{\mathbf{sets}}$: number of AV-exclusive link sets considered per network, γ: penetration rate range

N	$\hat{\alpha}$	β	n_{ens}	μ	p_{splice}	n_{paths}	n_{sets}	γ
100	0.5, 0.75, 1	1.5	50	3	0.75	10	5	$0.1, 0.3, \ldots, 0.9$

3. RESULTS AND DISCUSSION

Numerical experiments are carried out on three different network ensembles generated with different $\hat{\alpha}$ (see figure 2 for exemplar networks from each).

We first look at the total system cost for the different ensembles. We are interested in the cost improvements attained by the management scheme, thus we express the costs in relation to the UE and SO costs. We calculate the *normalised costs* for each network, averaging it across all the networks in each ensemble. The normalised cost for a network is

$$C_{\text{normalised}} = \frac{C(\mathbf{f}^{\text{ME}}) - C_{\text{SO}}}{C_{\text{UE}} - C_{\text{SO}}}, \tag{9}$$

where C_{UE} is the cost of UE, C_{SO} is the cost at SO, and $C(\mathbf{f}^{\text{ME}})$ is the cost of the flows resulting from the control scheme.

Figure 4 shows the normalised costs averaged across the networks of each ensemble for our three experimental ensembles.

As expected, costs are reduced with increasing penetration rate of AVs in the fleet. Even for small amounts of AVs ($\gamma = 0.1$), the mean falls at or below the UE cost. For low penetration rates, individual networks can exceed UE costs (as shown by the standard deviation), whereas for the ensemble with $\hat{\alpha} = 1$, the ensemble average lies above UE for several demand values. There can be a large variation within each ensemble, primarily for low penetration rate, which reflects that both the network structure and choice of AV-exclusive links are important. However, even for modest penetration rates, such as $\gamma = 0.3$, there are significant network performance improvements across the three ensembles for all values demand values. Even costs that are above the ensemble mean by one standard deviation remain below 0.75 and, for the $\hat{\alpha} = 1$ ensemble, it is closer to 0.5.

A major effect of changing the morphology of the networks is a variation in the error range. This is more evident for the $\hat{\alpha} = 0.75$ ensemble, which performs better in this regard, suggesting that the results are more consistent across networks that are constructed from lightly scrambled grids. The main difference between $\hat{\alpha} = 0.75$ and $\hat{\alpha} = 1.0$ is that, for the latter, nodes can be a lot closer together, which further breaks the regular block structure of the networks. This suggests that for very regular networks or for very random ones, the a-priori benefits the scheme might bring have higher uncertainty. However, by looking at the error ribbon for low penetration rates ($\gamma = 0.1$) and its overlap with the mean curves of even $\gamma = 0.5$, an appropriate choice of AV-exclusive links has the potential of effects similar to much higher penetration rates.

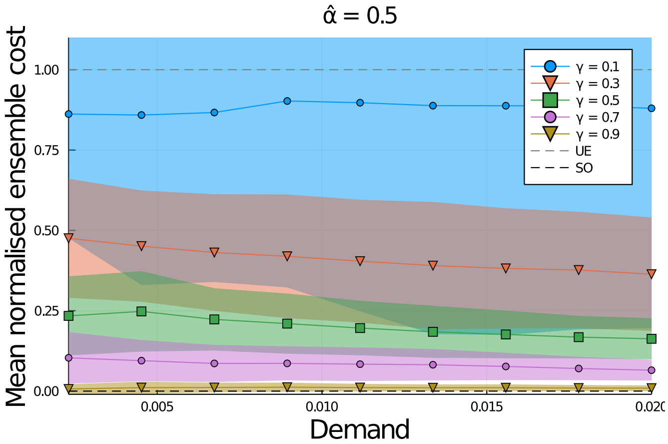
A final observation on the improvements is that, across all the demand range studied, the mean normalised costs remain fairly constant, which means that the performance of the scheme is stable to changes in demand of the network.

Figure 5 shows the per-vehicle costs for the three ensembles as functions of the AV penetration rate γ for low, medium, and high demand. This is compared to the per-vehicle cost at UE for each demand level. The behaviour is consistent for all the ensembles. Across the whole γ range, AVs experience higher per-vehicle costs than the HVs. For low penetration rates, the average per-vehicle cost experienced by AVs is larger than at UE. As γ increases, the costs eventually fall below the UE costs at levels between 60% and 80%. Higher demand levels experience this cross-over at larger penetration rates.

These results are in line with (Van Vuren et al., 1990) and (Espinosa Mireles de Villafranca, 2020), where no flow restrictions are imposed on streets. This means that, unfortunately, for the way we select the AV-exclusive links, the management scheme is not enough to reduce the AVs' costs enough to fall below HV level.

4. CONCLUSIONS

We presented a framework for traffic management in networks, where a proportion of the vehicles can be routed altruistically to improve system-wide travel times. In combination with the altruistic vehicles, a subset of network links on



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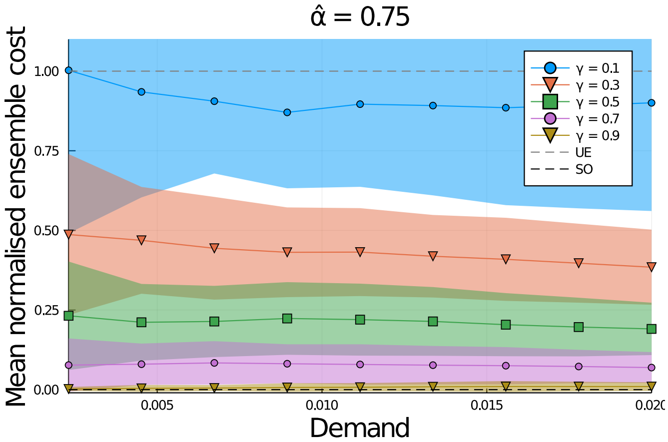

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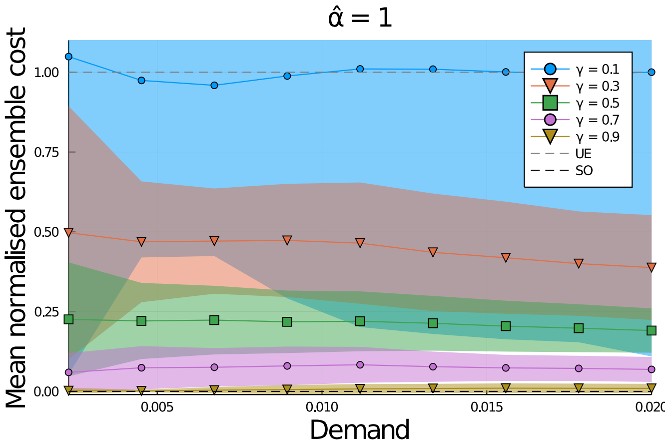
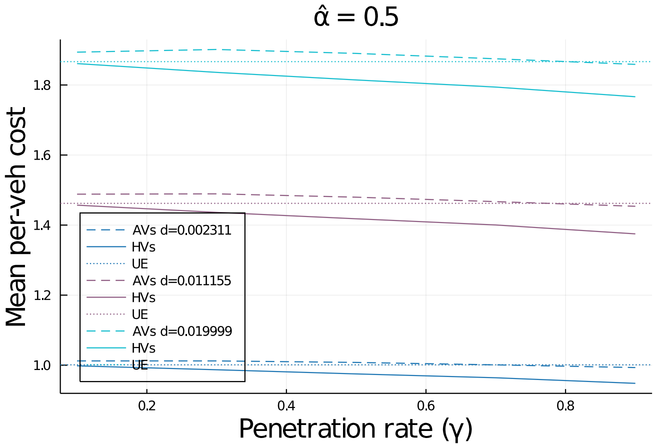
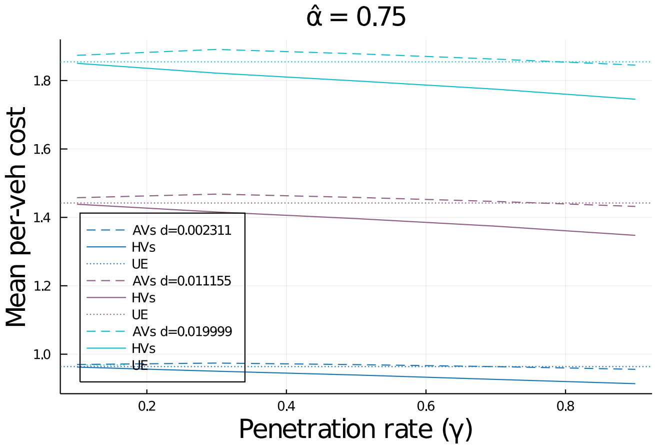


(c)

Figure 4: Normalised costs for three 50 network ensembles: (a) $\hat{\alpha} = 0.5$ (b) $\hat{\alpha} = 0.75$ (c) $\hat{\alpha} = 1.0$. The ribbons extend one standard deviation above and below the mean (except where cut off by the axes limits)



(a)



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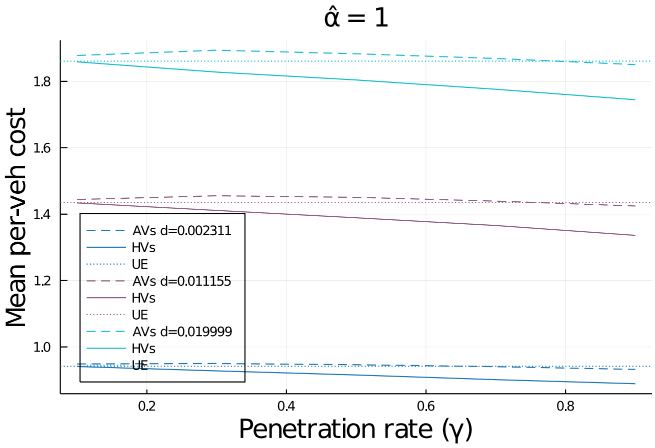

(c)

Figure 5: Per-vehicle costs as a function of γ for different ensembles: (a) $\hat{\alpha} = 0.5$ (b) $\hat{\alpha} = 0.75$ (c) $\hat{\alpha} = 1.0$. For the different d values, the per-vehicle costs are shown in comparison to the UE cost per-vehicle

which only AVs can travel are chosen. In our implementation, the choice is made from five competing subsets and the one with the best performance is chosen for the current demand and AV penetration rate on the network.

In terms of overall performance, the scheme is successful in reducing costs even for modest penetration rates (e.g., 30%). We do note, however, that the benefits can vary significantly from network to network. In our numerical experiments, the choice of AV-exclusive links is not optimised, while we merely provide a set of alternatives to avoid particularly bad samples. The idea behind this is to give a general idea of the potential benefits, whilst taking into account that in reality there can be a multitude of factors (e.g., practical, political) that might not allow the choice of the optimal set.

When considering per-vehicle costs, our results are consistent with other studies in the literature: AVs experience higher travel times than HVs. A large enough penetration rate guarantees lower costs than at UE, however, this does not occur until at least a 60% penetration rate. To understand whether the scheme is enough to make travel by AV attractive, differences in the value of time of the different classes would be needed to be taken into account, but falls beyond the scope of this paper.

We have shown that across a range of network structures, a management scheme like the one we propose can improve system performance. The way we select AV-exclusive links can serve as a benchmark for performance improvements, leaving the door open for more sophisticated strategies. An important question that remains unanswered is: can simple interventions lead to AVs experiencing better travel times than HVs?

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