A nested recursive logit model for route choice analysis

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Discrete choice models are generally used for analyzing route choices in real networks. There are two main challenges associated with estimating the parameters of such models. First, the choice sets are unknown. Second, path utilities may be correlated due to physical overlap in the network. In order to address the first issue, either choice sets of paths can be sampled and utilities corrected for the used sampling protocol, or the recursive logit (RL) model (Fosgerau et al., 2013) can be used. These approaches are both based on the multinomial logit (MNL) model and hence cannot adequately address the second challenge. This paper presents an extension of the RL model that relaxes the independence from irrelevant alternatives (IIA) property and the resulting model therefore addresses both the two aforementioned challenges.

Before describing the methodology we give a brief literature review focusing on the gap that this research aims to fill. The choice set generation problem has received a lot of attention in the literature and there are numerous algorithms designed for generating choice sets of paths. Typically, they are based on some kind of repeated shortest path search with a varying generalized cost. Frejinger et al. (2009) note that depending on the choice set definition, significantly different parameter estimates can be obtained for a same model and a same data set. They therefore argue path utilities should be corrected for the sampling of alternatives and they propose such a sampling correction for the MNL model. Guevara and Ben-Akiva (2013) propose a correction for generalized extreme value models but it has not yet been used in a route choice context. These approaches can be used to obtain consistent parameter estimates based on samples of alternatives but it is unclear how to use the models for prediction and no correction for this case has been derived. The RL model is based on the same underlying assumption as the model based on sampled alternatives, namely, that the universal choice set is composed of all paths in the network. Contrary to the path based model, RL is easy and fast to use for prediction. RL is however equivalent to a MNL model over all paths in the universal choice set and hence suffers from the IIA property.

There are several models proposed in the literature to model correlation between paths (e.g Bekhor et al., 2002, Chu, 1989, Frejinger and Bierlaire, 2007, Vovsha and Bekhor, 1998) but these models do to take into account that the choice set is sampled. Those that correspond to a generalized extreme value model can be corrected (Guevara and Ben-Akiva, 2013) while it is unclear how to do it for the mixed logit models. Similar to the MNL models using sampled alternatives, it is also unclear how to correct utilities when using the models for prediction.

The nested RL model presented here fills a gap in the literature because (i) the IIA property is relaxed and the model (ii) does not require sampling of paths and (iii) it is straightforward to use for prediction. The following presentation of the model assumes some knowledge of the RL model and we refer the reader to Fosgerau et al. (2013) for more details. Travellers choose links in a sequential manner maximizing the instantaneous utility of the next link (action) a given his/her current location in the network (link/state k). In the RL model the instantaneous utility is $u(a|k) = v(a|k) + \varepsilon(a)$ where the random terms are independently and identically distributed extreme value type I. For NRL these utilities are $u(a|k) = v(a|k) + \mu_k\varepsilon(a)$ where $0 < \mu_k \leq 1$ is a state specific scale parameter. The key part of the RL model is the value function $V(a)$ capturing the expected maximum utility from the sink node of the action link a to the destination d. In RL they correspond to logsums and are a solution to a system of linear equations. For the NRL this is not the case. Indeed, the value functions given by the Bellman equation are

$$V(k) = \begin{cases} \mu_k \ln \sum_{a \in A} \delta(a|k) e^{\frac{(v(a|k)+V(a))}{\mu_k}} & \forall k \in A \\ 1 & k = d \end{cases} \tag{1}$$

where A is the set of links in the network and $\delta(a|k)$ equals one if a and k are sequential links. We solve this fixed point problem by value iteration.

The probability of choosing a given state k is given by the MNL model

$$P(a|k) = \delta(a|k) \frac{e^{\frac{1}{\mu_k}(v(a|k)+V(a))}}{\sum_{a' \in A(k)} e^{\frac{1}{\mu_k}(v(a'|k)+V(a'))}} = \delta(a|k) e^{\frac{1}{\mu_k}(v(a|k)+V(a)-V(k))}$$

and the likelihood of a path $\sigma = \{k_i\}_{i=0}^{I}$ is

$$P(\sigma) = \prod_{i=0}^{I-1} e^{\frac{1}{\mu_{k_i}}(v(k_{i+1}|k_i)+V(k_{i+1})-V(k_i))}. \tag{2}$$

In this case the ratio of two path probabilities do not depend solely on their respective utilities and the IIA property does therefore not hold. Note that there are as many scale parameters μ_k as there are links in the network and we cannot estimate all of them. We therefore assume that the scale is a function of some link attributes each associated with a parameter to be estimated. We derive analytical expressions for the gradient and Hessian and we show that they can be effectively approximated by solving the system of linear equation. This is important for the estimation but the detailed expressions are quite involved and presented in the full paper.

We also note that solving (1) by value iteration is computationally much more expensive than for the RL case. In order to improve the estimation performance, we propose a simply method for solving the non-linear equation with dynamic accuracy. More precisely, the value iteration method is started with some low number of iterations and we increase the number of iterations as the estimated parameters are close to the optimal solution. We observed that choosing the initial value functions that satisfy the system of linear equation given by the RL model greatly improves the convergence speed.

For the numerical results we use the Borlänge data, same as the one used in Fosgerau et al. (2013). Moreover, we use the same deterministic utility specification as them except that we estimate the parameter associated with u-turns and we have two additional parameters for the scale parameter. The utility is

$$u(a|k;\beta,\omega) = v(a|k;\beta) + V(a) + \mu_k(\omega)\epsilon(a) \quad (NRL2)$$

and in this case $\mu_k(\omega) = \exp(\omega_{TT}TT_k + \omega_{LF}LF_k)$ where TT is travel time and LF link flow. For the sake of comparison we also estimate a NRL model with fixed scales (NRL1) where the scales are proportional to link travel time. The estimation results are presented in Table 1. The NRL2 specification significantly improves the model fit while keeping the parameter estimates from the RL model stable. We note that fixing scale proportional to link travel time is not as a good idea.

Parameters	RL	NRL1	NRL2
$\widehat{\beta}_{TT}$	-2.494	-0.836	-2.572
Rob. Std. Err.	0.098	0.201	0.099
$\widehat{\beta}_{LT}$	-0.933	-0.303	-0.904
Rob. Std. Err.	0.030	0.09	0.030
$\widehat{\beta}_{LC}$	-0.411	-0.127	-0.344
Rob. Std. Err.	0.013	0.040	0.014
$\widehat{\beta}_{UT}$	-4.459	-1.453	-4.442
Rob. Std. Err.	0.114	0.379	0.133
$\widehat{\omega}_{TT}$	-	-	0.307
Rob. Std. Err.	-	-	0.276
$\widehat{\omega}_{LF}$	-	-	-0.946
Rob. Std. Err.	-	-	0.088
$LL(\widehat{\beta})$	6303.9	6298.8	6211.4

Table 1: Estimation results for real data

This is still ongoing research and in the final paper we plan to present an illustrative example showing the correlation pattern resulting from the NRL model. We will also investigate the link to the classic nested logit model.

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References

Bekhor, S., Ben-Akiva, M., and Ramming, M. Adaptation of logit kernel to route choice situation. *Transportation Research Record*, 1805:78–85, 2002.

Chu, C. A paired combinatorial logit model for travel demand analysis. In *Proceedings of the fifth World Conference on Transportation Research*, volume 4, pages 295–309, Ventura, CA, 1989.

Fosgerau, M., Frejinger, E., and Karlström, A. A link based network route choice model with unrestricted choice set. *Transportation Research Part B*, 56:70–80, 2013.

Frejinger, E., Bierlaire, M., and Ben-Akiva, M. Sampling of alternatives for route choice modeling. *Transportation Research Part B*, 43(10):984–994, 2009.

Frejinger, E. and Bierlaire, M. Capturing correlation with subnetworks in route choice models. *Transportation Research Part B*, 41(3):363–378, 2007.

Guevara, C. A. and Ben-Akiva, M. E. Sampling of alternatives in multivariate extreme value (MEV) models. *Transportation Research Part B*, 48(1):31 – 52, 2013. ISSN 0191-2615.

Vovsha, P. and Bekhor, S. Link-nested logit model of route choice Overcoming route overlapping problem. *Transportation Research Record*, 1645: 133–142, 1998.