Identifying the Distribution of Value of Travel Time with a Monotonic Nonparametric Estimator

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Abstract

Traditional discrete choice models, even with extra error components, are weak on identifying the value of travel time (VTT) free of distributional assumption and getting finite moment of VTT distribution. Fosgerau (2006) and Börjesson et al. (2012) used several nonparametric and semi-parametric techniques to identify the cumulative density function (c.d.f.) of VTT. This kind of methods has advantages including: 1) free of distributional assumption; 2) can be used for screening the parametric distribution candidates; 3) mean VTT can be calculated in a nonparametric way. However, we should note that in these studies: 1) the VTT calculated nonparametrically might be biased if the estimated c.d.f. violates the monotonicity; 2) Single-bounded bid design was used, which was proved to be less efficient than double-bounded (i.e. the following bid is adaptively adjusted according to the response to last bid) bid design (Hanemann et al., 1991),.

This paper proposes a nonparametric estimator with monotonic constraints, and applies it to binary choice model with parameters lying on willingness-to-pay space. Subsequently a case study using stated preference data based on double-bounded bid design is carried out for obtaining the VTT distribution for Japanese car users.

Supposing that respondents need to decide whether to trade cost for time while making decision on two parallel routes, we can rearrange the alternative order such that $y = 1$ always indicates the slower and cheaper alternative being chosen and $y = 0$ otherwise. Then it is possible to switch the indirect utility function from preference space to WTP space via reparameterization as (Fosgerau, 2006) as follows.

$$\begin{aligned} y &= 1\{\beta_t t_0 + \beta_c c_0 > \beta_t t_1 + \beta_c c_1\} \\ &= 1\{\beta_t/\beta_c < -(c_0 - c_1)/(t_0 - t_1)\} \end{aligned} \tag{1}$$

Let $w = \beta_t/\beta_c$ denotes individual's unobserved, heterogeneous, true VTT which is i.i.d. with c.d.f. F_w, and $v = -(c_0 - c_1)/(t_0 - t_1) = \Delta c/\Delta t$ denotes the VTT bid, then we can rewrite the above formula simply as $y = 1\{w < v\}$. We also assume $w \perp (\Delta t, \Delta c)$ so that $P(y = 1) = P(w < v) = F_w(v)$. If we further assume $w = \bar{w} + u$ and u subject to i.i.d. logistic distribution, then $P(y = 1) = P(\bar{w} - v < u)$, which gives rise to a binary logit in WTP space. However, we do not want impose any distributional assumption except adding a measurement error term η that holds $E(\eta|v) = 0$. Subsequently we have $y = F_w(v) + \eta$ and $E(y|v) = F_w(v)$. The VTT c.d.f. is therefore able to be identified by regressing y on v nonparametrically as long as v extends over the support of F_w.

To estimate F_w, we first consider a Nadaraya-Watson Estimator (NW estimator) for its simplicity. In the standard one-dimensional NW estimator (see Li and Racine, 2007), let $\{v_i, y_i\}_{i=1}^n$ be the sample of n pairs of independent variable and dependent variable, the expected value of y conditional on v is given by the weighted average of y_i in its neighbourhood

$$\hat{F}_w(v) = E(y|v) = \sum_{i=1}^{n} \frac{K(\frac{v-v_i}{h})y_i}{\sum_{i=1}^{n} K(\frac{v-v_i}{h})} \tag{2}$$

where K is the kernel function that weights distance between v and v_i and the bandwidth h decides the neighbourhood's size, namely the number of y_i to be averaged on. Here we use leave-one-out cross-validation to decide the optimal bandwidth as following.

$$h^* = \arg\max_h \frac{1}{n} \sum_{i=1}^{n} \{y_i - \hat{F}_w^{h,-i}\}^2 \tag{3}$$

where $\hat{F}_w^{h,-i}$ is the leave-one-out estimator. Given the noise, however, this estimator does not ensure monotonicity for the estimated function.

To impose the monotonic increasing constraint, we generalize the NW estimator by imposing additional weights $p = \{p_1, ..., p_n\}$ to $\{y_1, ..., y_n\}$ as Hall and Huang (2001). From another viewpoint, p can also be seen as a probability distribution on $\{v_1, ...v_n\}$. Therefore, the function estimate is given by

$$\hat{F}_w(v) = E(y|v) = n \sum_{i=1}^{n} p_i \frac{K(\frac{v-v_i}{h})y_i}{\sum_{i=1}^{n} K(\frac{v-v_i}{h})} \tag{4}$$

p is initialized as $\{1/n, ..., 1/n\}$, namely uniform distribution, so that it collapses to the standard NW estimator as long as p keeps initial value. Otherwise, the function estimate departs from unconstrained one. Consequently by minimizing the L_2 distance between vector p and $\{1/n, ..., 1/n\}$ subject to $\hat{F}'_w(v) \geq 0$, it is available to get p^* that give rise to a function estimate that is closest to unconstrained $\hat{F}_w(v)$ and holds monotonicity. $p_i \geq 0$ and $\sum_i p_i = 1$ also need to be satisfied because p is treated as a probability distribution. In addition, two extra constraints need to be imposed since we are identifying a c.d.f. which can neither exceed 1 nor be lower than 0. Therefore, finally the constrained quadratic optimization problem is summarized as

$$
\begin{aligned}
\min_{p_1,\dots,p_n} & \sum_{i=1}^{n} (n^{-1} - p_i)^2 \\
s.t. & \sum_{i=1}^{n} p_i = 1 \\
& p_i \geq 0 \\
& \hat{F}'_w(v) \geq 0 \\
& \hat{F}_w(v) \geq 0 \\
& \hat{F}_w(v) \leq 1
\end{aligned}
\tag{5}
$$

The stated preference (SP) experiment for this study consists of a series of binary route choice situations where alternatives are constructed by only two attributes: time and cost. First, the time t and cost c distance (times fuel cost per km plus toll) in respondents' last trip are asked and treated as the reference point in light of prospect theory (Tversky and Kahneman, 1992; De Borger and Fosgerau, 2008). Pivoted on the reference trip, the alternatives are constructed on two choice types: willingness-to-pay (WTP), willingness-to-accept (WTA). Specifically, Δt is computed as the product of reference time and a random number within the range of [10%,30%], then Δc is calculated by $\Delta t * v$, where v is the random VTT-bid uniformly taken from 4 strata within [1,80] for the odd question and the median between boundary[1] and last VTT-bid.Subsequently, Δt and Δc are assigned to alternatives interacting with reference time and cost, such that respondents always need to trade off time and cost. Each respondent needs to make decision in 9 choice situations, including four choice situations for each choice type and a dominant choice situation.

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[1]lower bound if the response to last question is choosing the slower and cheaper alternative and upper bound otherwise

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