Efficient calibration of stochastic traffic simulation models

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Introduction

This paper develops a mathematically sound methodology for the calibration of traffic simulation models. As explained in the following, the approach sets itself apart from the existing literature in that it combines response information from a complex traffic micro-simulation with a tractable analytical approximations of the highly non-linear traffic flow dynamics, allowing for the application of efficient and robust gradient-based optimization routines to the calibration problem.

Typical traffic micro-simulations capture travel demand in terms of time-dependent origin/destination (OD) matrices, from which they sample individual trip makers. Usually, a probabilistic route choice model is then applied to select a route for each trip maker, and a mesoscopic or microscopic traffic flow model is used to propagate the trip makers' vehicles through the network. If the route choice model is congestion-dependent, then route choice and traffic flow simulation are iterated until a stochastic fixed point of mutual consistency is attained (Barcelo; 2010).

Microscopic simulations enable detailed representations of reality, and as such they are built around data-intensive model systems. This renders the automatic calibration of micro-simulations a difficult and practically relevant problem. A largely unresolved methodological challenge in this context is the formulation of tractable measurement equations that link available surveillance data from the real transport system to the model parameters one wishes to calibrate.

Most existing approaches to this problem rely on black box optimization routines that exploit problem structure hardly beyond numerical differentiations. Examples of deployed methods are SPSA (Spall; 1992), the Kalman Filter in various guises (extended, limiting, unscented), and derivative-free search techniques. These approaches have in common that they require large computation efforts, mainly because they exploit problem structure not beyond a numerical linearization of the measurement equation. See Antoniou (2004); Balakrishna (2006) for two representative monographs and Ben-Akiva et al. (2012) for a comprehensive literature review.

The recent work of Flötteröd et al. (2011, forthcoming, 2012) constitutes a notable exception in that it solves a concrete instance of the calibration problem (estimation of route choice parameters from traffic counts) in a very efficient manner based on an *analytical* approximation of the measurement equation's gradient. The present work builds on this approach and complements it by casting the problem into the computationally efficient framework of simulation-based optimization (Osorio; 2010).

Methodology

Formal problem statement

To make the proposal concrete, we focus in the following on the calibration of travel demand parameters from network flows. A generalization to other data sources and parameters to be calibrated is is part of the planned research effort.

Each OD pair $n = 1 \ldots N$ is connected by one or more routes. The set routes in OD pair n is denoted by C_n. The total travel demand within OD pair i is written as d_n. The probability that a traveler in OD pair n selects a route $i \in C_n$ is written as $P_n(i \mid \mathbf{x}, \boldsymbol{\beta})$ where $\mathbf{x}$ represents the network attributes (in particular, travel times) that characterize the alternative routes and $\boldsymbol{\beta}$ is a vector of coefficients (to be estimated) that guide the choice process.

Let $\delta_{i,t}^{a,k}(\mathbf{x})$ be one if a traveler having departed on route i in time interval t enters link a in time interval k, and zero otherwise. This parameter depends in the presence of congestion on the travel times contained in $\mathbf{x}$. The flow (in vehicles) across link a in time step k can then be written as

$$q_{a,k} = \sum_{n=1}^{N} d_n \sum_{i \in C_n} \delta_{i,t}^{a,k}(\mathbf{x}) P_n(i \mid \mathbf{x}, \boldsymbol{\beta}) \tag{1}$$

Since the network travel times contained in $\mathbf{x}$ depend in turn on the network flows defined here, this equation is circular and can in general only be solved iteratively. In a traffic microsimulation, these iterations can be interpreted as a learning process over subsequent days, where in each day all trip makers select a route according to the most recent network conditions $\mathbf{x}$, followed by a simulation of the corresponding vehicle flows through the network, which in turn yields updated network conditions.

Let $y_{a,k}$ be the number of vehicles counted in reality on link a in time step k. A natural non-linear least squares formulation of the calibration problem is then to minimize the following objective function:

$$Q(\boldsymbol{\beta}) = \sum_{a,k} (y_{a,k} - q_{a,k})^2 \tag{2}$$

where $q_{a,k} = q_{a,k}(\boldsymbol{\beta})$ due to (1). The difficulty of this problem is owed to the complexity of constraint (1), which is not available in closed form but is represented only procedurally through the traffic microsimulation.

This project proposes to solve Problem 2 by embedding structural information that *analytically* approximates the main components of (1). We expect this analytical information to significantly enahnce the computational efficiency of the calibration algorithm, and ultimately to allow us to even solve challenging real-time calibration problems.

Simulation-based optimization approach

To start off, we focus on the calibration of the route choice parameters. We assume an analytical closed-form expression for the corresponding route choice model is given, $P_n(i \mid \mathbf{x}, \boldsymbol{\beta})$. We propose to derive an analytical approximation of the System of Equations (1).

The main idea is to derive an analytical approximation that combines information from the simulator (i.e., this is the traditional approach) with information from an analytical macroscopic and computationally efficient traffic model. Ideas along these lines have been used to efficiently address large-scale urban traffic management problems while using inefficient yet detailed microsimulators (Osorio and Chong; 2012; Osorio and Nanduri; 2012; Chen et al.; 2012). This combination will be formulated based on metamodel (also called surrogate model) ideas from the field of simulation-based optimization (SO), as in Osorio and Bierlaire (2010).

Metamodel SO techniques iterate over two main steps which are depicted in Figure 1. Firstly, the metamodel, m, is constructed based on a sample of simulated observations. Secondly, it is used to perform optimization and derive a trial point (i.e., a route choice parameter value). The performance of the trial point can be evaluated by the simulator, which leads to new observations. As new observations become available the accuracy of the metamodel can be improved (Step 1), leading ultimately to better trial points (Step 2).

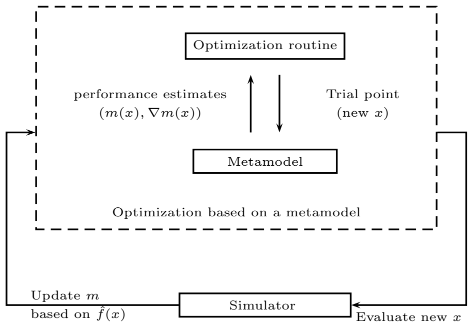

Figure 1: Metamodel simulation-based optimization methods. Adapted from Alexandrov et al. (1999).

Traditional techniques fit metamodel based only on simulated information. The main novelty proposed by this project is to combine this simulated information with analytical approximations derived by tractable analytical macroscopic traffic models. The metamodels will then be embedded within a state-of-the-art SO algorithm (Osorio and Bierlaire; 2010), which is based on the use of derivative-free trust region algorithms (Conn et al.; 2009).

The analytical traffic model used will be based on queueing network theory. Recent models developed by the PI's combine traffic flow theory ideas with queueing theory ideas in order to develop probabilistic analytical and differentiable models of urban traffic (Osorio and Flötteröd; 2012; Osorio et al.; 2011; Osorio and Bierlaire; 2009a,b). Recent work has also derived techniques to further enhance the scalability and tractability of these models (Osorio and Chong; 2012; Osorio and Wang; 2012).

These models will provide an analytical approximation of how route choices $P_n(i \mid \mathbf{x}, \boldsymbol{\beta})$ for the population of travelers affect the main network attributes (e.g., flows and travel times). This is a challenging problem because the approximated mapping involves the highly nonlinear and stochastic *network loading map* of path flows on network conditions, comprising in the micro-simulation context all difficulties that come along with real traffic flow dynamics in urban networks (including, e.g., multi-lane flows, spill-back, flow interactions in complex intersections). The recent doctoral dissertation of Frederix (2012) makes clear that this is anything but a solved problem. By deriving analytical and efficiently computable structural information to the calibration algorithm, we expect to derive highly efficient calibration techniques. We will evaluate the performance of this novel technique by calibrating route choice parameters for simple toy networks, as well as for a large-scale microscopic model.

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