Route choice model estimation with simulated attributes

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Problem statement

The increasing availability of detailed, individual-level mobility data enables the estimation of complex discrete choice models of travel behavior. The corresponding choice contexts, however, may be difficult or impossible to observe and hence need to be imputed. In this work, we focus on the effect of uncertainty in the modeling of travel time attributes for very simple route choice models. Such uncertainty results in particular if the decision maker is exposed to a changing and/or stochastic environment and the analyst is unaware of the concrete information acquisition and learning protocol implemented by the decision maker.

The analysis is constructed around a synthetic two-route network, where a large number of travelers select their routes according to a logit choice model that only accounts for congestion-dependent travel times. Synthetic observations, consisting of the number of travelers choosing either route per day, are used to analyze the characteristics of two different estimators of model parameters (specifically, the travel time coefficient) and of attribute uncertainties (specifically, a parametrized travel time covariance matrix). Travel times are assumed to be unavailable to the analyst and hence need to be imputed in the estimation. In brief summary, the joint estimation of travel times and their (co)variances appears feasible.

Two-routes example

Consider a microscopic traffic assignment model that equilibrates congestion-dependent route choice. There are N decision makers that select from identical route choice sets $C_n = C = \{A, B\}$. The choice of traveler n is written as $i_n \in C$. Letting

$$n_i = \sum_{n=1}^{N} \mathbf{1}(i_n = i) \tag{1}$$

be the flow on route $i \in \{A, B\}$, its sole attribute is its congestion-dependent travel time

$$t_i = a_i n_i + b_i \tag{2}$$

where $\mathbf{1}(\cdot)$ is the indicator function and a_i, b_i are known parameters of the congestion-dependent network loading mechanism. Traveler n assigns to route i the random utility

$$U_{ni} = \mu \mathbf{E}\{t_i\} + \varepsilon_{ni} \tag{3}$$

where μ is a coefficient for the expected (mean) travel time $\mathbf{E}\{t_i\}$ and ε_{ni} is a stochastic error term, leading to a random utility model that assigns the probability $P_n(i \mid \mathbf{t}; \mu)$ to the event of traveler n choosing route i given travel times $\mathbf{t} = (t_A, t_B)$ and travel time coefficient μ.

We now assume that the analyst is uncertain about the travelers' perception of travel time because of recent fluctuations in the network conditions and the unavailability of a model describing the travelers' perception and learning of these fluctuations. Letting the random variable

$$Z_{ni} = \mathbf{E}\{t_i\} + \eta_{ni} \tag{4}$$

represent the (to the analyst unknown) perception of traveler n of route i's travel time, with η_{ni} being a stochastic error term, one obtains the model

$$\begin{aligned} U_{ni} &= \mu Z_{ni} + \varepsilon_{ni} \\ &= \mu \mathbf{E}\{t_i\} + (\mu \eta_{ni} + \varepsilon_{ni}). \end{aligned} \tag{5}$$

The challenge in estimating this model is the fact that the error terms η_{ni} may have a fairly complicated dependency structure. First, there may be dependency across alternatives. This dependency is at least in parts owed to the fact that the information processed by the traveler consists of previously observed travel times, which were generated by a physical process of network flow propagation with strong interactions between routes. Second, there may be dependency across individuals because all decision makers were exposed to and hence have observed the same physical environment.

Methodology

This work is based on the assumption that real decision makers implement some kind of learning and exploration protocol on which they base their behavior, but it is not assumed that the analyst is able to model this protocol. The analyst should hence model the corresponding attributes of alternatives as random variables, including a realistic dependency structure.

This is possible by generating these attributes within a stochastic process framework that in other work has been used as an approximation of actual learning. In the present work, however, the purpose of this stochastic process approach is to simulate stochastic attributes with a realistic dependency structure, without any claim to model human learning.

The following stochastic process model is considered.

1. Give each traveler n some initial information $\mathbf{Z}_n = (Z_{A,n}, Z_{B,n})$ about route travel times.

2. Repeat the following process until stationarity is attained.

(a) Let each traveler select a route according to the model $P_n(i \mid \mu, \mathbf{Z}_n)$.

(b) Compute the resulting route flows and route travel times $\mathbf{T} = (T_A, T_B)$.

(c) Update each traveler's travel time information according to

$$\mathbf{Z}_n \leftarrow \alpha \mathbf{Z}_n + (1 - \alpha)\mathbf{T}$$

with $\alpha \in [0, 1)$ controlling the degree of smoothing in this update process.

Assuming, for simplicity, (i) that all travelers share identical information and (ii) that the mapping of current knowledge $\mathbf{Z}$ on (choices on flows on) resulting travel times $\mathbf{T}$ is approximated well by a linear model, one obtains

$$\mathrm{E}\{\mathbf{Z}\} = \mathrm{E}\{\mathbf{T}\} \tag{6}$$

$$\mathrm{VAR}\{\mathbf{Z}\} = \frac{1-\alpha}{1+\alpha}\mathrm{VAR}\{\mathbf{T}\}. \tag{7}$$

That is, the stochastic travel time update process (i) does not introduce a bias into the simulated travel time perception, (ii) exhibits the same correlation structure as the unfiltered travel times, and (iii) has a level of randomness that is parametrized by the parameter α, ranging from almost zero (for $\alpha \to 1$) to that of the unfiltered travel times (for $\alpha = 0$). This stochastic process model can hence be used to simulate route choice decisions based on imputed and congestion dependent travel times with a parametrized covariance structure. The simulation-based nature of this approach needs to be stressed, in that the travel time covariance matrix is not explicitly specified but results from the iterative congestion feedback loop.

Experiments

Synthetic data is generated by a verbatim implementation of the stochastic process model described before. This clearly is a simplification (it was previously explicitly assumed that the analyst is unaware of the concrete learning protocol implemented by the travelers), but it is a useful first step to investigate the identifiability of the model.

A homogeneous population of $N = 1000$ travelers is assumed, all of which select a route in every day based on a logit-form choice model $P_n(i \mid \mathbf{Z}, \mu)$. Different setting of μ and α lead to different stationary dynamics of this process. Figure 1 shows several histograms over $R = 1000$ independent realizations of the stationary flow on route A, indicating oscillations for low smoothing coefficients α in conjunction with high travel time coefficients μ.

Two different estimators of the parameters (η, α) are then investigated. Both are simulation-based in that they incorporate the previously described stochastic process model in order to capture travel time variability. Their objective functions, however, are different. The first estimator is based on a nonlinear least squares objective function, while the second estimator uses a maximum-likelihood formulation. The main difference of the two estimators is that the least squares estimator implicitly assumes a univariate flow distribution, whereas the maximum likelihood estimator also allows for multivariate distributions.

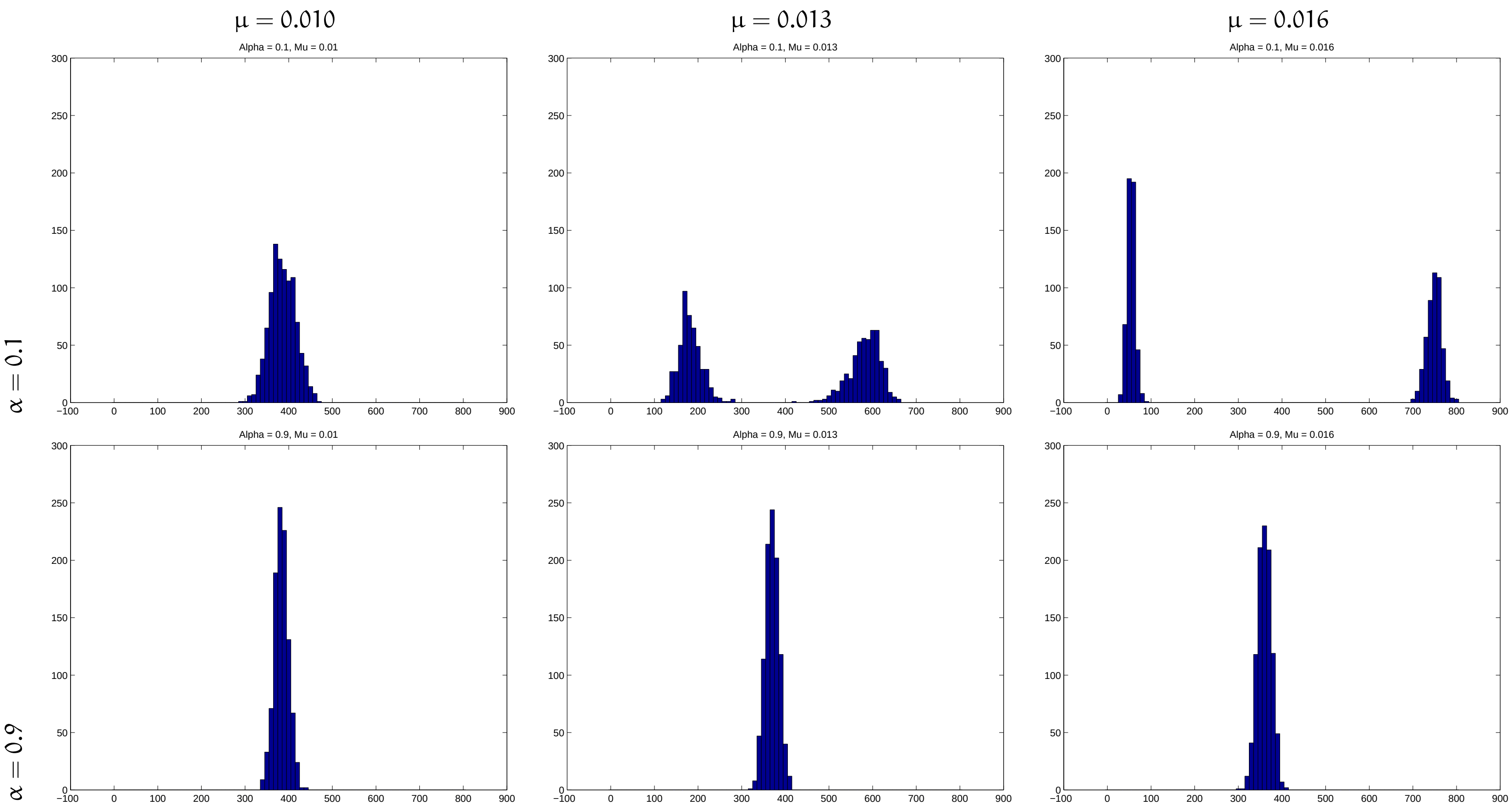

Figure 1: Histogram of stationary traffic flow on route A

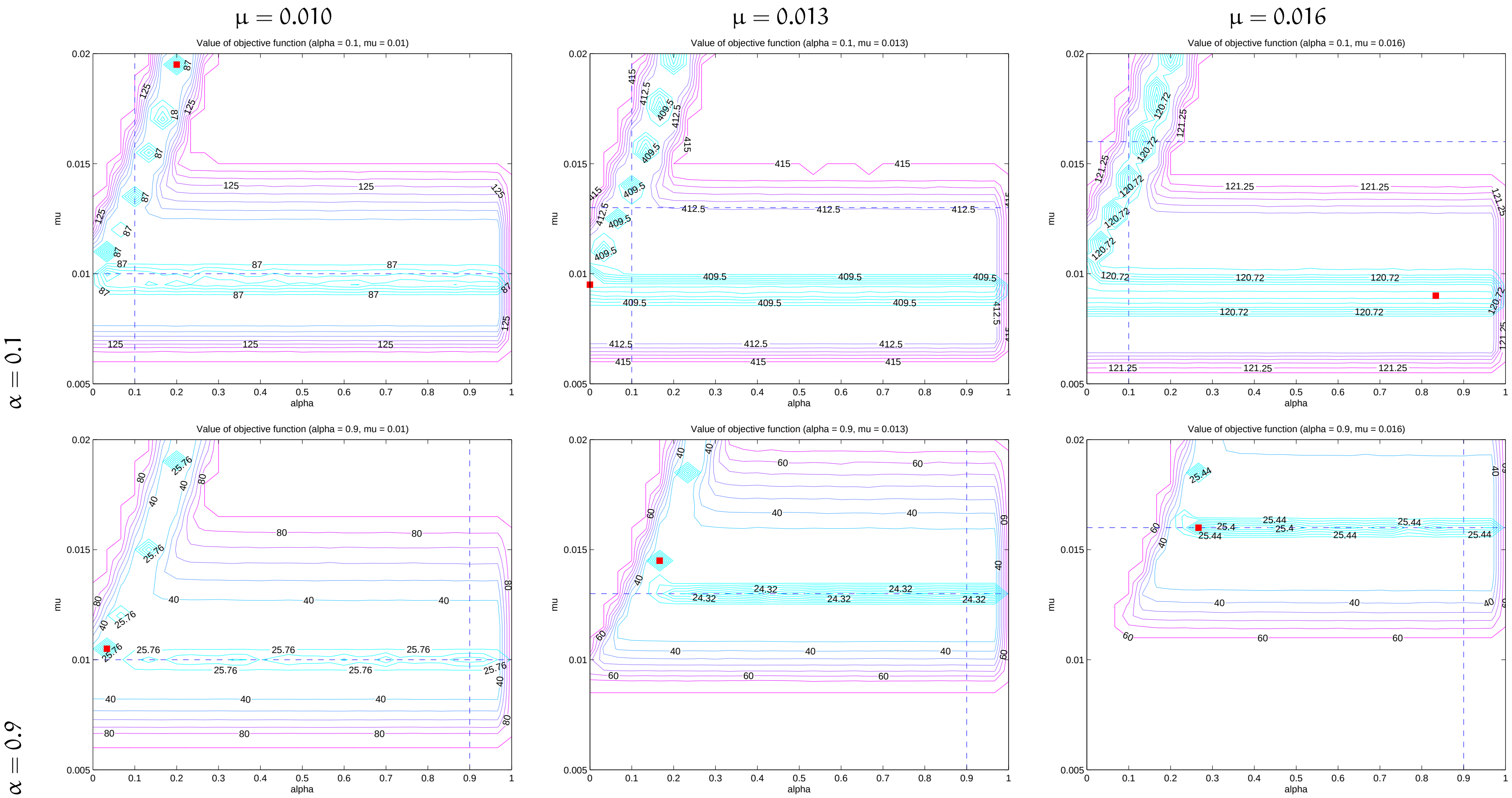

Figure 2: Least Squares objective functions

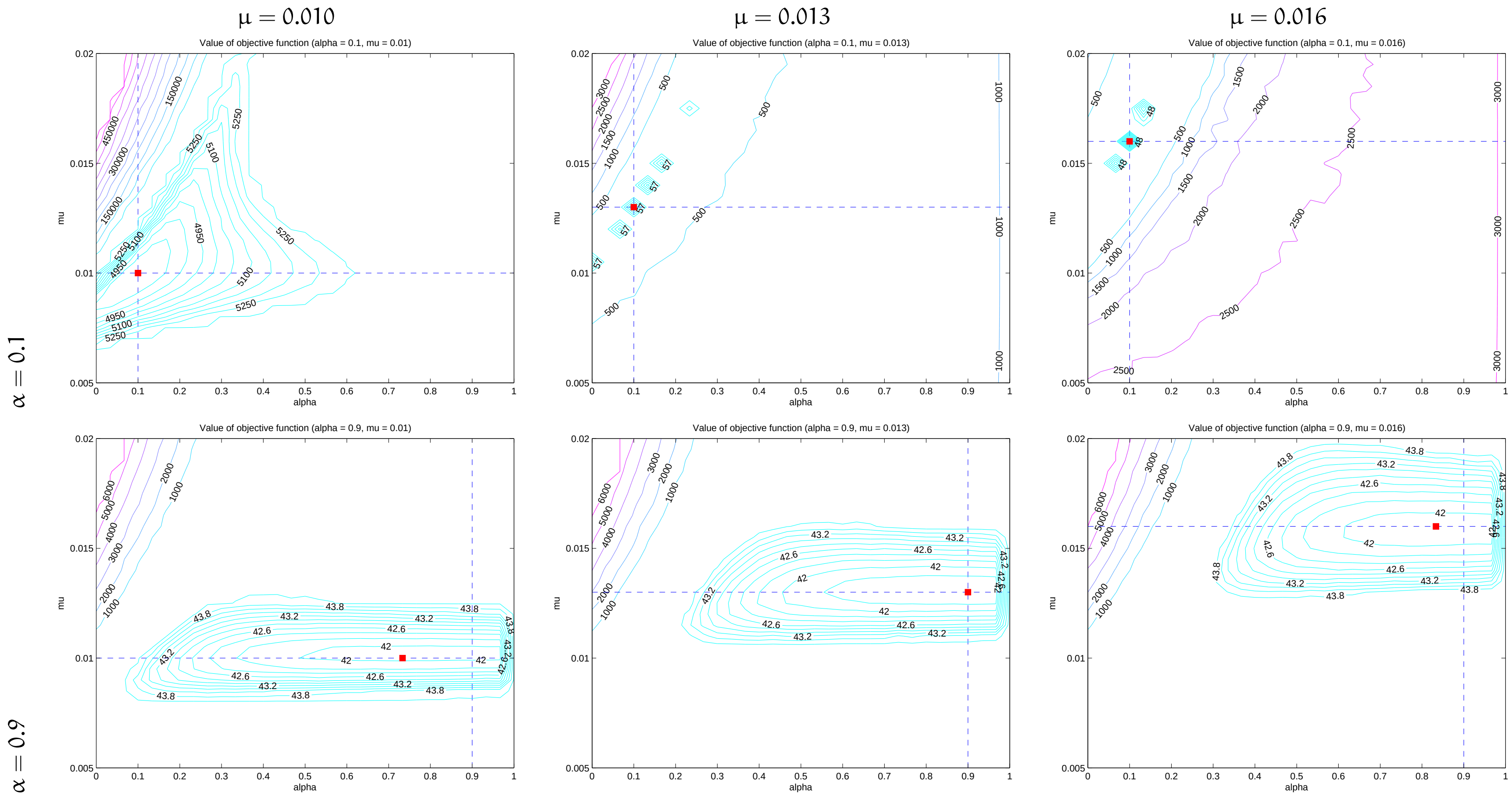

Figure 3: Log-likelihood functions

These properties are also reflected in the objective functions, shown in Figure 2 and 3, the latter displaying the negative log-likelihood function. In all Figures, the blue dashed cross represents the true parameter values (based on which the simulated observations were generated), and a red square is put on the grid point with overall smallest objective function value. Due to the relatively high computational cost, the functions are plotted on a coarse grid that is somewhat rugged in both cases. While a comprehensive discussion of these and further results is postponed to the full paper, the following observations can already be made based on this first visual impression.

1. The least squares objective function is relatively ill-behaved, with long valleys and multiple optima. A search algorithm would have difficulties in identifying a global optimum, and the alpha parameter appears hardly identifiable.

2. The maximum-likelihood estimator is better behaved, with multiple optima occurring only in the case of oscillating system behavior. The α parameter appears identifiable, and it appears possible to recover the μ parameter without bias.