The Effect of Signalized Intersections on Dynamic Traffic Assignment Solution Stability

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The on-going interest in dynamic traffic assignment (DTA) models in general, and continuous-flow (analytic) models in particular, stems from the need to better understand the various dynamic effects that can be observed in transportation networks. Representation of traffic signal delays, which are partly due to random queuing effects, is a particular challenge in these models.

In this research we examine a continuous-flow analytic DTA model with Markov representation of traffic signals. The dynamic network loading component of the DTA model is based on the authors' previous work [1], in which trajectories and anticipated arrival order to nodes are used to achieve consistency between kinematic waves (physical queue) representation of single link traffic behavior and flow propagation along routes. The traffic signal component consists of: a non-stationary cycle-to-cycle Markov chain for the evaluation of queue length randomness; deterministic delay consideration; within cycle behavior treatment; and a scheme for integration with the network level model. Numerical examples illustrate that overall the model behaves properly, and captures interesting impacts of random queuing traffic signal delays on route choice and network level DTA solutions.

A primary focus of this research is solution stability that may be strongly influenced by the DTA model specification, and in particular, by the way time is discretized to produce a computable DTA solution. The challenge of discretizing continuous dimensions is well known in many scientific domains when Finite Elements Methods (FEM) are used such as: elasticity and structural analysis in mechanical engineering, civil engineering and aeronautical engineering; simulations in physics; differential equations in mathematics; etc.

The discretization into finite elements through FEM enables to convert continuous analytic equations which may be difficult or impossible to solve in close form into a finite set of numerical equations with a computable solution. This discretization may lead to numerical artifacts as can be observed even in very simple FEM dealing with single dimension first-order ordinary differential equation (ODE) of the form $y(t) = f(Y(t))$ where $y(t) \equiv Y^{'}(t)$, with a given initial value, $Y(t_0) = Y^0$. In the finite element approximation a time step duration, δt, is chosen, and the solution is determined for integer increments of the time step, $Y^n = Y(n\delta t)$. It is assumed that the derivative is constant within each time step, that is $y(t) = y^n$ for $n\delta t \leq t \leq (n+1)\delta t$. As a result

$$Y^{n+1} = Y^n + y^n \delta t. \tag{1}$$

In this finite element representation there is an inherent inconsistency between the discretized values of Y^n that correspond to points in time and the discretized values of y^n that correspond to time intervals. Due to this inherent inconsistency, the ODE $y(t) = f(Y(t))$ cannot be satisfied, in general, at all times t. Euler suggested two alternatives to deal with this difficulty. In the *forward Euler method*, the derivative in the interval is associated with value at the beginning of the interval:

$$y^n = f(Y^n), \tag{2}$$

while in the *backward Euler method*, the derivative in the interval is associated with value at the end of the interval:

$$y^n = f(Y^{n+1}). \tag{3}$$

In either case we obtain a set of finite difference equations that can be solved numerically. In some ODEs, referred to as *stiff equations*, spurious oscillations exist in the forward Euler solution.

The discretization challenge in DTA has a similar nature. To illustrate the similarity the continuous dynamic user equilibrium (DUE) can be formulated as follows. Let $d_{pq}(\tau)$

be the total flow rate from origin p to destination q, for departure time τ, $h_r(\tau)$ the proportion of OD flow on route r at departure time τ, $C_r(\tau)$ the cost of travel along route r for travelers departing at time τ, and $C^*_{pq}(\tau)$ be the minimum OD cost for departure at τ. The conservation of flow requirement is defined as:

$$H_0 = \left\{ \mathbf{h} : \sum_{r \in Z_{pq}} h_r = 1 \quad \forall pq \right\}, \tag{4}$$

where Z_{pq} is the set of all routes from origin p to destination q. For a given vector of route costs, $\mathbf{C}$, define the set of all minimum cost assignments as:

$$H(\mathbf{C}) = \left\{ \mathbf{h} \in H_0 : \begin{array}{ll} h_r > 0 & C_r = C^*_{pq} \\ h_r = 0 & C_r \geq C^*_{pq} \end{array} \quad \forall r \in Z_{pq}, \forall pq \right\}. \tag{5}$$

The continuous DUE problem is then to find the proportions by departure time for all routes such that:

$$\mathbf{h}(\tau) \in H(\mathbf{C}(\tau)) \quad \forall \tau. \tag{6}$$

In the discrete DUE problem we assume that route demands are constant within each departure time interval so that $h_r(\tau) = h^n_r$ for $n\delta t \leq \tau \leq (n+1)\delta t$. Since travel times are not constant within an interval there is a need to determine which travel time will be associated with an interval. As in the Euler method, here as well, there is more than one alternative for discretization. In the forward alternative, which is referred here as *equilibrium condition lag* 0 ($ECL = 0$), the travel time associated with each interval is the travel time computed at the beginning of the interval ,that is:

$$\mathbf{h}^n \in H(\mathbf{C}^n) \quad \forall n. \tag{7}$$

where $\mathbf{C}^n = \mathbf{C}(n\delta t)$. In the backward alternative, which we call *equilibrium condition lag* 1 ($ECL = 1$), the travel time associated with each interval is the travel time computed at the end of the interval:

$$\mathbf{h}^n \in H(\mathbf{C}^{n+1}) \quad \forall n. \tag{8}$$

We conducted a set of computational experiments characterized by three dimensions: 1) traffic signal representation; 2) route choice interval (RC); and 3) equilibrium condition lag (ECL). For traffic signal representation we focus on two approaches, exit capacity and Markov chain. The first one is a simplistic baseline approach in which traffic signals are represented by a fixed exit capacity on the relevant link, aimed at capturing the dynamic queue accumulation component of delay only. The second approach aims to treat both the steady-state deterministic and random components of the delay as well as its dynamic queue accumulation component in an integrated framework. The experiments were conducted using the network illustrated in figure 1, which is a variant of the Nguyen and Dupuis network [4].

Results using the exit-capacity approach are presented in figures 2 and 3, showing time-dependent assignment proportions and travel times for the 5 alternative routes of OD B-C. Each subplot shows a combination of route choice interval ($RC = 2\delta t$, $RC = 4\delta t$, $RC = 10\delta t$) and equilibrium condition lag (ECL=0, ECL=1). These results confirm previous findings (e.g. [3]) that backward Euler methods (ECL=1) lead to the most stable solutions (see the assignment solution for OD B-C in figure 2).

Figures 4 and 5 present similar results for the case signals are represented by the proposed Markov approach. Comparing for example the solutions with ECL=1 and $RC = 10\delta t$ between figures 2 and 4, we can see that traffic signal representation may have non-trivial effects on network-level behaviors. See [2] for additional discussion of these effects. In terms of stability, ECL=1 performs better than ECL=0, as we saw previously. However, in some of the cases ($RC = 2\delta t$) results are not stable even with ECL=1. In an attempt to improve stability we examined the option of ECL=2, but it did not lead to better results. Stability improved quite considerably with longer route choice interval ($RC = 10\delta t$). Additional aspects of this and other example are presently under investigation.

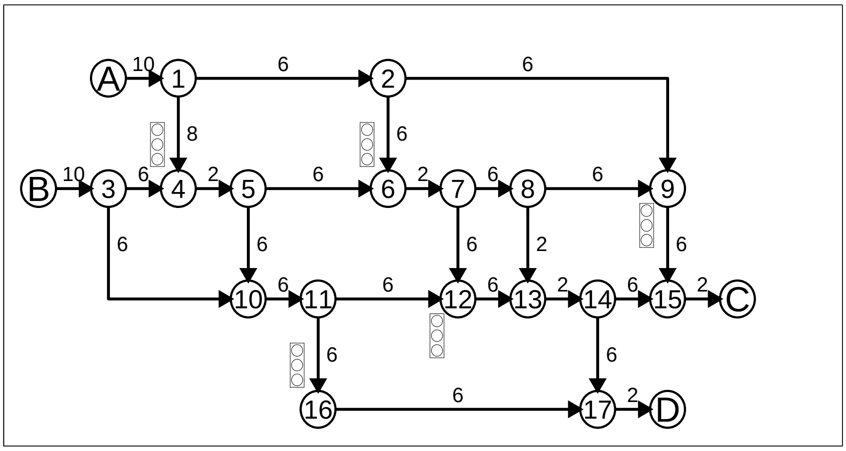

Figure 1: Nguyen and Dupuis's Network. Values along the links indicate lengths in km.

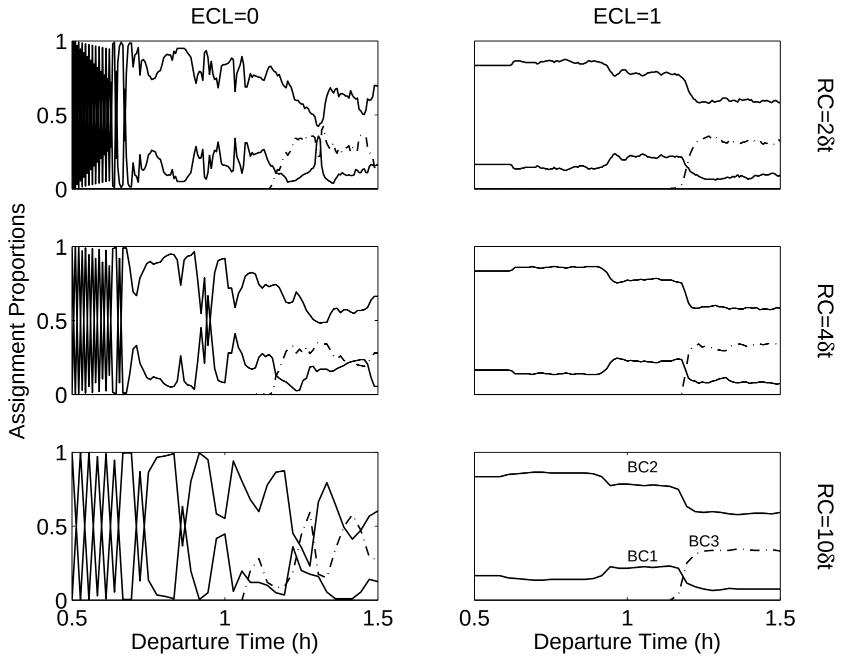

Figure 2: Assignment proportions for Nguyen and Dupuis's network with exit capacity mechanism, OD $B-C$

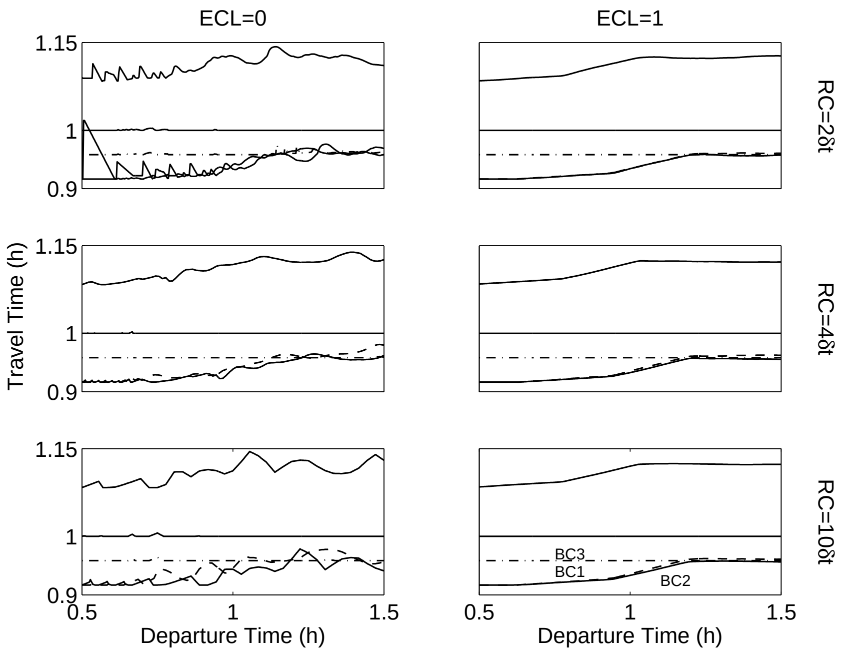

Figure 3: Travel times for Nguyen and Dupuis's network with exit capacity mechanism, OD $B-C$

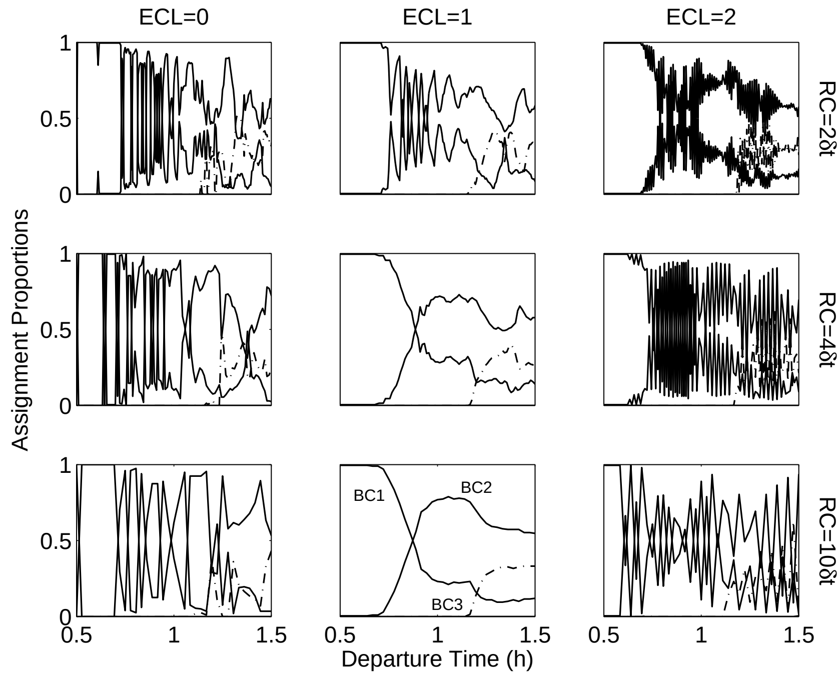

Figure 4: Assignment proportions for Nguyen and Dupuis's network with integrated signal delay evaluation method, OD $B - C$

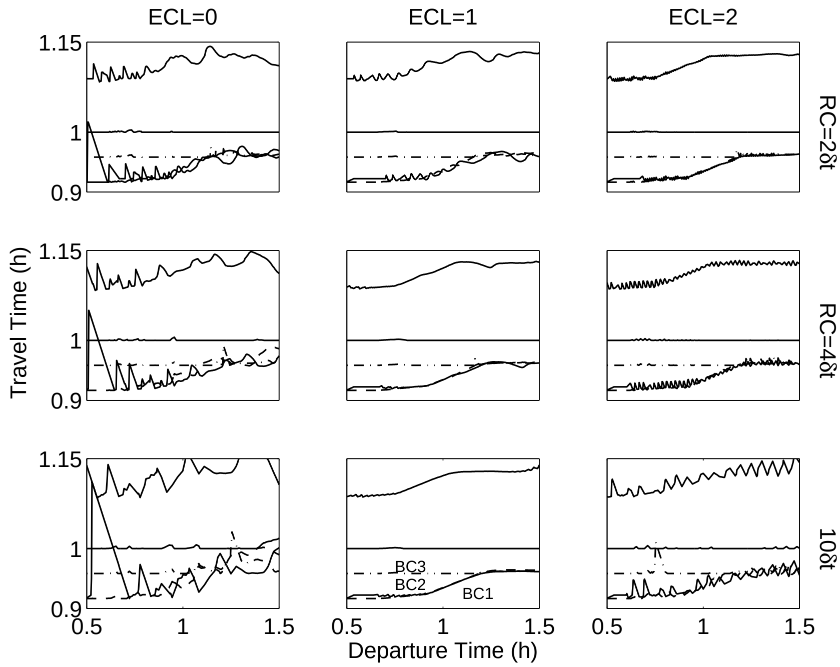

Figure 5: Travel times for Nguyen and Dupuis's network with integrated signal delay evaluation method, OD $B - C$

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