# Solving Bi-objective Traffic Equilibrium Based on Time Surplus Maximisation

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The conventional user equilibrium (UE) approach to model traffic assignment assumes that all users have the same objective, i.e. to minimise their travel time or generalised cost, which usually represents a linear combination of time and monetary cost. In tolling analysis, however, different users might behave very differently in response to the toll. To address this problem, [1] proposed a bi-objective approach to model equilibrium in a tolled network. Users are assumed to have two criteria: (1) to minimise travel time; and (2) to minimise toll cost. As defined in [1], under bi-objective user equilibrium (BUE) condition, traffic arranges itself in such a way that no individual trip maker can improve either their toll or travel time or both without worsening the other component by unilaterally switching routes. With this approach, however, there might exist infinitely many solutions. Several heuristics to find some of the solutions are presented in [2].

As discussed in [3], not all solutions satisfying the BUE condition occur in practice. Therefore, it is important to incorporate more specific assumptions on the route choice behaviour in the model. This can be achieved by applying the concept of time surplus maximisation bi-objective user equilibrium (TSMaxBUE) as proposed in [3]. Time surplus is defined as the maximum time a user is willing to spend minus the actual time spent. The maximum time a user is willing to spend is modelled as an indifference curve – a linear or non-linear function that depends on the path toll. Under the TSMaxBUE condition, traffic arranges itself in such a way that no individual trip maker can improve his/her time surplus by unilaterally switching routes, or alternatively, all individuals are travelling on the path with the highest time surplus value among all the efficient paths between each origin-destination (O-D) pair. In this way, the two objectives of minimising travel time and toll can be transformed into a single objective, i.e. to maximise time surplus. The solution to the TSMaxBUE model will naturally satisfy the BUE condition as well.

To solve the TSMaxBUE model, we consider two cases separately: one user class (i.e. all drivers are assumed to have the same indifference curve that might differ only for different O-D pairs) and multiple user classes (i.e. there are several groups of drivers corresponding to different indifference curves).

We show that the TSMaxBUE model with one user class can be solved using the time-based traffic equilibrium (TBEQ) model as described in [4] as long as the indifference curve is continuous, strictly decreasing and non-negative. We thereby propose a practical application of the TBEQ model. In the TBEQ model the travel time and flow-independent toll are combined into a generalised time via a non-linear non-negative strictly increasing function that represents "the time equivalent of money". In [4] an equivalent optimisation formulation for the TBEQ model was also established. The existence of this equivalent program is a very important advantage from a practical point of view because optimisation techniques can be used to solve the TBEQ model and hence the TSMaxBUE model with a single user class. Moreover, it allows us to estimate how far away from the equilibrium a current solution is.

In [4] the authors implemented aggregated and disaggregated simplicial decomposition methods. However, other optimisation-based approaches can also be applied. Here we propose to apply the path equilibration approach as presented in [5] and more advanced techniques such as the projected gradient [6] and the gradient projection [7]. Previously these methods were applied to solve the conventional traffic assignment problem with travel time as the objective and were reported to have a very promising performance. This motivates us to test the performance of adopting these methods to solve the more complicated

TSMaxBUE model. We note that path costs are not additive in the TSMaxBUE problem and must be evaluated at the path level, so that path flows must be used as the explicit variables. The non-additivity of the path cost is one of the main difficulties in the adaptation of optimisation techniques. Because of this conventional shortest path algorithms cannot be used. Instead, the BUE condition states that users must travel on efficient paths with respect to travel time and toll. Paths with maximal time surplus will be efficient paths, hence approaches based on bi-objective shortest path algorithms must be applied, see [8].

The TSMaxBUE model with a single user class is, however, not practical as it means that all users have the same indifference curve between time and toll. A much more challenging task appears when we consider the TSMaxBUE model with several user classes. In this case no equivalent optimisation formulation exists, which implies that other techniques have to be used to find its solutions. These techniques can be derived from variational inequality or non-linear complementarity (NCP) formulations, or directly based on the definition of equilibrium. The most promising approach that we adopt here is the NCP formulation that can then be transformed into a global optimisation problem as proposed in [9].

The BUE condition is conceptually different from the approaches based on aggregation of objectives. It is inspired by the multi-objective definition of optimality and allows multiple solutions (potentially infinitely many of them). We propose the TSMaxBUE model as a possible way to represent route choice behaviour in tolled road networks. In case of one user class it can be transformed into the TBEQ model which can be solved by optimisation-based algorithms. To solve the TSMaxBUE model we adopt some path-based algorithms used for conventional traffic assignment, compare their performance and study how the solution space depends on the chosen indifference curve. In case of multiple user classes generally it is not possible to derive an equivalent optimisation formulation. Therefore, we propose to use the NCP formulation to solve the TSMaxBUE model.

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