Computational study of path-based cross entropy method for solving simulation-based dynamic traffic assignment problem

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1 Introduction

The development of solution algorithms for simulation-based dynamic traffic assignment has been widely studied in recent years. This issue is important since dynamic traffic assignment (DTA) has a variety of applications in planning, route guidance and motorway traffic control for large-scale transportation network. The DTA problem aims to obtain user equilibriums with respect to traffic supply and demand variations. This problem can be formulated as a variational inequality, non-linear complementarity or fixed point problem [1]. Many solution algorithms have been proposed in the past including method of successive averages (MSA) [2], gap function based projection approach (GP)[3], dynamical system approach [4], and many others [5]. The path-based algorithms have been widely applied for solving large-scale network assignment problem. The reader is referred to [6, 7] for detailed review.

Recently, the cross entropy (CE) method has been proposed for solving general static/dynamic traffic assignment problem [8, 9]. The CE method is a stochastic optimization technique for solving combinatorial optimization problems. This method optimizes the flow shifts on more attractive routes until dynamic user equilibrium is achieved. Previous studies showed that the CE algorithm can efficiently solve multiple user class static/dynamic traffic assignment problems. However, for large-scale network assignment problem, the enumeration of all possible paths for each origin and destination is problematic. The development of efficient path generation algorithm is needed for applying the CE method for realistic network applications.

For this issue, this paper proposes a new path-based CE algorithm for solving large-scale network assignment problem. A computational study is implemented for solving static and dynamic simulation-based traffic assignment problems. The performance of the proposed CE algorithm is compared with the MSA and GP algorithms to verify its convergence speed and solution accuracy.

2 Solution algorithms

The dynamic user equilibrium problem is formulated as a non-linear minimization problem [3]. The dynamic network loading model is based on the extended point queue model [10], which satisfies generic first order macroscopic node models [11]. The reader is referred to [8,9,10] for more detail description.

We detail the proposed path-based algorithm as follows.

Step 1 (Initialization): Compute k-shortest paths for each OD pair based on free-flow travel time. Set iteration index w=0. Let $\tilde{R}_{hk}^{w}$ be choice path set with respect to departure time interval h and OD pair k at iteration w. $\tilde{r}_{hk}^{w}$ denotes the shortest path in $\tilde{R}_{hk}^{w}$. Initialize the uniform probability distribution for $\tilde{R}_{hk}^{w}$. Travellers randomly select a departure time instant within related departure time interval.

Step 2 (Dynamic network loading): Move travelers into the network according to his/her departure time and path choice. When travellers arrive at his/her destinations, compute the experienced travel cost. Update average time varying link travel time with respect to discretized entering time of link. Compute shortest paths for each OD pair k and departure time interval h based on modified Dijkstra's shortest path algorithm. Let $\tilde{r}_{hk}^{w+1}$ denote the shortest paths currently found with respect to h and k. Compute the value of gap function, which measures how experienced travel cost is far from the idealized shortest path cost.

Step 3 (Choice probability update): Update the path choice probability as

$$p_{hkr}^{w+1} = p_{hkr}^{w} \frac{e^{-\hat{C}_{hkr}^{w}/\gamma_{hk}^{w}}}{\sum_{s\in\tilde{R}_{hk}^{w}} p_{hks}^{w} e^{-\hat{C}_{hks}^{w}/\gamma_{hk}^{w}}},\ \forall r \in \tilde{R}_{hk}^{w}, \text{if } \tilde{r}_{hk}^{w+1} \in \tilde{R}_{hk}^{w} \tag{1}$$

$$p_{hkr}^{w+1} = \frac{e^{-\hat{C}_{hkr}^{w}}}{\sum_{s\in\tilde{R}_{hk}^{w}} e^{-\hat{C}_{hks}^{w}}}, \forall r \in \tilde{R}_{hk}^{w+1} = \tilde{R}_{hk}^{w} \cup \tilde{r}_{hk}^{w+1}, \text{if } \tilde{r}_{hk}^{w+1} \notin \tilde{R}_{hk}^{w}, \tag{2}$$

where

$\hat{C}^{w}_{hkr} = \frac{\overline{C}^{w}_{hkr}}{\overline{C}^{w}_{hk}}$ is normalized travel cost with respect to h, k and r with $\overline{C}^{w}_{hkr} = \frac{1}{|\Omega^{w}_{hkr}|} \sum_{m \in \Omega^{w}_{hkr}} C^{w}_{m}(t)$ and $\overline{C}^{w}_{hk} = \frac{1}{|\Omega^{w}_{hk}|} \sum_{m \in \Omega^{w}_{hk}} C^{w}_{m}(t)$.

Ω_{hkr} : set of travelers with respect to h, k and r

$C_m(t)$: traveler m's experienced travel cost when departing at time t

γ^{w}_{hk} is the control parameter with respect to h and k resulting from the solution of the following minimization problem:

$$\text{Min } \gamma^{w}_{hk} \text{ subject to } \sum_{r \in \widetilde{R}^{w}_{hk}} | p^{w+1}_{hkr} - p^{w}_{hkr} | \leq \theta^{w}, \tag{3}$$

where $\theta^{w} = \kappa / w$ is a numerical divergent series such that the flow adjustment converges. κ is a positive constant. w is an iteration index. Set $w := w + 1$

Step 4 (Stop criteria): When $w = w^{\max}$ or the resulting probability updates stabilize, stop; otherwise goto Step 2.

3 Computational results

We present the computational results for static and simulation-based dynamic traffic assignment problems. Four realistic-size problem instances are tested, selected from the *Transportation Network Test problems* [12] The performance of the algorithms is compared with respect to different network size and levels of congestion (parameterized by time varying travel demand and link flow capacity).

The average convergence of the CE, MSA and GP algorithms for solving static traffic assignment is illustrated in Fig.1. It shows that the ordered average solution accuracies are CE >=MSA>DP (Fig. 1). Note that the average gap function value is calculated with respect to three congestion levels for the selected test instances.

For simulation-based dynamic traffic assignment problem, we report the computational result on the selected largest network (Anaheim network of 416 nodes, 914 arcs and 1406 OD pairs) It indicates that the proposed CE method outperforms the MSA and GP methods (Table 1). The first result of the proposed CE algorithm is very promising although the time dependent shortest path computation is very time-consuming. However, it shows that the CE method provides satisfactory convergence accuracy to user equilibrium. Further study is needed concerning fast path choice set generation algorithm to improve the convergence speed of the proposed CE method.

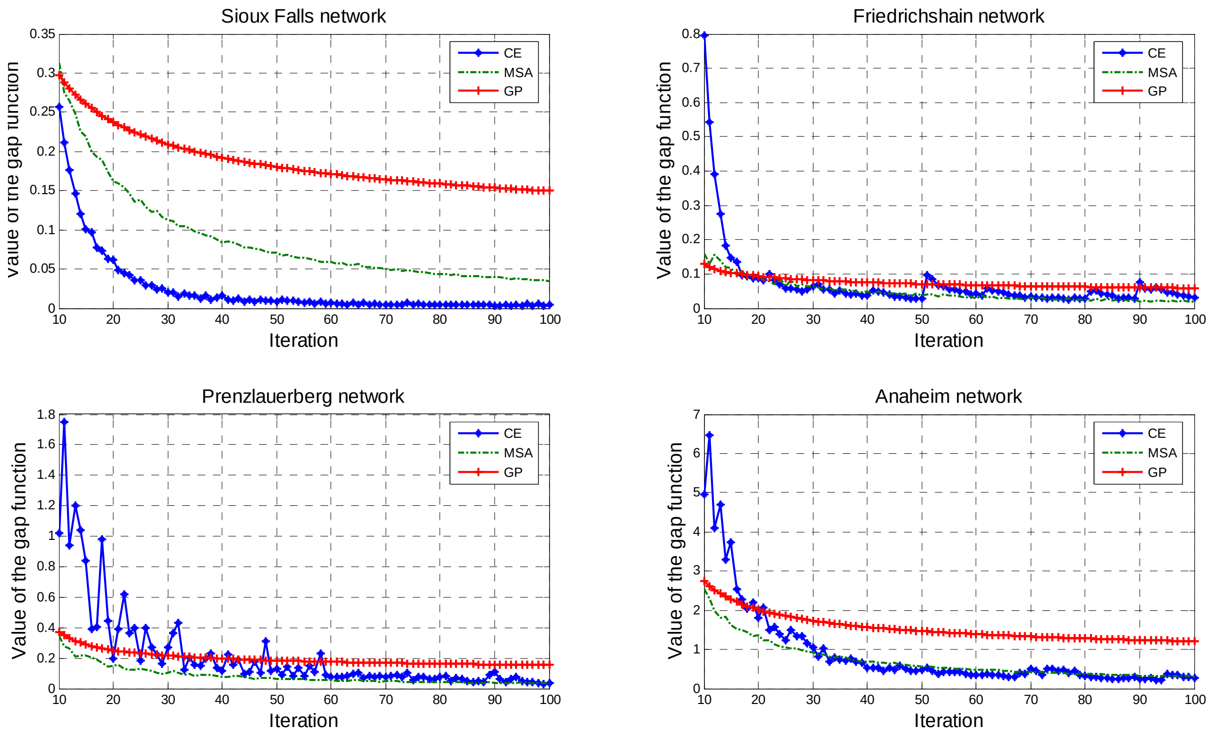

Fig. 1 Average convergence results of CE, MSA and GP algorithms

Table 1 Performance of the CE, MSA and GP algorithms on Anaheim network (416 nodes, 914 arcs and 1406 OD pairs)

	CE		MSA		GP	
N	Gap	Time (sec.)	Gap	Time (sec.)	Gap	Time (sec.)
673 ($d_k/100$)	0.0105	212.7	0.0117	224.5	0.0115	240.5
4669 ($d_k/20$)	3.1149	1052.3	3.13428	1027.5	3.40	1021.3

Remark: 1. the result is based on the best performance of three runs
2. d_k is the number of trips on Anaheim network from the instance dataset

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