Day-to-day dynamic route choice and signal control in traffic networks

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1 Introduction

It is well known that the efficiency of signal control strategies in traffic networks is affected by driver route choice behavior. The seminal works of Allsop [1] and Gartner [7] showed the close interaction between traffic control and traffic assignment, followed by a large number of studies that extensively explored this combined assignment and control problem [12]. While most of the literature focuses on the final equilibrium state, an important stream of research keeps attention to the day-to-day route choice adjustment processes in relation to traffic control. Such models enable one to consider the long-term behavior of dynamic traffic systems and to evaluate scenarios also in unstable equilibrium situation. This study explores this research stream and particularly it analyzes: 1) its model representation; and 2) equilibrium stability of the dynamic systems under different modeling and system behavior assumptions.

Our analysis focuses in particular on two lines of research, by Smith [8,9,10,11] and Cantarella [3,4,5,6] respectively, which are especially relevant for this research problem. In a number of aspects these two modeling approaches differ significantly, and in others they clearly show analogies and some overlapping characteristics. Smith began his studies in the late-1970s with consideration on devising an applicable traffic control policy which can be fitted in the traffic assignment model under congested traffic conditions. He proposed a well-known policy, the P0 control policy, which is found to have agreeable properties of convergence and is shown to maximize the overall network capacity. In his later studies, P0 is compared with other control policies, showing its superiority in efficiency and stability. Very recently, Smith included this policy in a day-to-day dynamic assignment model, thus to study the driver re-routing and signal re-timing in an idealized dynamical model.

In the early-1990s, Cantarella and Cascetta developed a unified framework for modeling dynamic processes and equilibrium in transportation networks [4]. The aim is to represent the realistic day-to-day learning process of travelers as well as to formulate conditions for stability properties. Cantarella later applied this modeling framework, in combination with daily re-optimization of signal timings, to the evaluation of the effects of traffic signal setting on urban network planning. Although taking different starting points, i.e. Smith focused on control policies to later refined users' response to signals on a day to day basis; while Cantarella began with a general model for day-to-day updating of costs and flows and then extended this framework to include signal setting optimization, they came to an analogous discussion on signal control and route choice in day-to-day dynamic models, yet showing seemingly little overlaps.

The motivation and essential contribution of this paper is therefore to compare the two different approaches, and systematically identify commonalities and methodological differences within a general framework of the combined signal control and route choice problem, able to comprise both theories as well as allowing for further extensions of the problem.

2 A general model for the day-to-day dynamic processes

In this section we summarize the main characteristics and differences between the key studies by the two authors, and introduce a framework that is able to unify and extend them.

2.1 Route choice adjustment and signal setting adjustment

Both Smith and Cantarella proposed methodologies for solving the signal control and route choice problem considering day-to-day dynamic adjustments. Comparing the two approaches, we identify three key issues in describing the dynamic systems with both route choice adjustment and signal setting adjustment.

- *Travel cost and cost updating factor*
- *Flow updating factor*
- *Signal updating factor*

The *cost updating process* may depend only on current traffic states, or also take into account historical costs in travel experience. Accordingly, the *flow updating process* may depend only on current cost difference, or also take into account previous route choices. Finally *signal updating process* can be derived from optimization or signal setting criteria, e.g. fixed-time or responsive signals; it can depend analogously on current traffic states, or also take into account and adjust previous timing plans.

In Smith's approach, the mathematical properties of uniqueness and stability are highlighted by carefully choosing the route swapping and signal setting criteria, which take into account only the latest travel cost. A route swapping rule updates flows according to the current cost difference between pairwise alternative routes, and a signal setting rule updates signal timings according to the difference of predefined traffic pressures, as defined in the P0 control policy [9].

In Cantarella's approach, travelers perceive costs based on both latest travel cost and past experience. Flow update takes into account travelers' knowledge on travel cost and previous choice habit, i.e. travelers reconsider yesterday's choice. Signal timings are either fixed or responsive during the process, in which the signal update depends on current traffic states and previous timings. Appropriate behavioral parameters and signal settings that are constrained by the equilibrium stability conditions can ensure modeling realism and stable solutions.

2.2 A general model

In the full paper, the following general modeling framework is proposed and extensively analyzed:

$$
\begin{aligned}
C^t &= \varphi(C^{t-1}, f^{t-1}, g^{t-1}) \\
f^t &= f^{t-1} + \alpha\phi^t \\
g^t &= g^{t-1} + \beta\psi^t
\end{aligned}
$$

In which C^t represents the travel cost information at day t, described as a function of previously observed cost information, flows f^{t-1} and signals g^{t-1}; ϕ^t is flow changing rate with a weight α; Ψ^t is signal changing rate with a weight β.

The realization of updating factors fits within different approaches, for example dealing with flow updating represented as expression (1) fits within Cantarella's approach; while expression (2) fits within Smith's approach. The two approaches are casted and compared within the general modeling framework (Table 1).

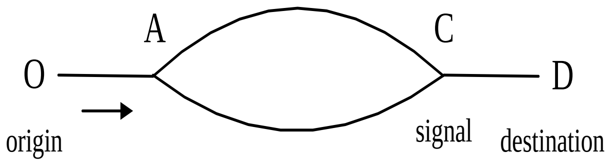
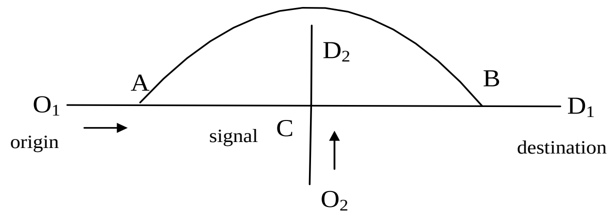
$$
\phi^t = \begin{cases} \mathrm{F}(C^{t-1}) - f^{t-1} & (1) \\ \sum_{OD}\sum_{(r,s)} f_r^{t-1}[C_r^{t-1} - C_s^{t-1}]_+ \Delta_{rs} & (2) \end{cases}
$$

Here, $[x]_+=\max\{x,0\}$, F(.) refers to relationship between flows and costs, Δ_{rs} is a vector with -1 in the *r*th place and +1 in the *s*th place, and 0 otherwise.

In a study of stability of the equilibrium assignment problem via a dynamical systems approach, Watling [13] showed the link between stability in continuous time systems (such as that employed in Smith's work) and in discrete time systems (such as that employed in

Cantarella’s work), without consideration on the signal effects during the updating process. Under the general modeling framework, the relation of flow updating and signal updating in these two approaches can be investigated, allowing for the extension of Watling’s study to also include the signal updating process.

Table 1 Updating processes of the two approaches

	Smith's approach	Cantarella's approach
	Main notations c: travel cost; f: flow; g: green signal; d: signal delay; s: saturation flow rate; k: constant; c(.): cost function; d(.): delay function; f(.): flow function; $g_{SS}(.)$: signal setting function; α, β, γ: flow, cost and signal updating weights	
Travel cost	$c = c(f,g) = c_l(f) + d(f,g)$ $c_l(f) = K + Af$, running time, K and A are constants $d(f,g)$, delay cost, generalized Webster formula	$c = c(f,g) = c_l(f) + d(f,g)$ $c_l(f)$ running time: Davidson cost-flow function $d(f,g)$ waiting time: Webster two-term function
Cost update	$c^t = c_l(f^{t-1}) + d(f^{t-1}, g^{t-1})$	$c^t = \beta c(f^{t-1}, g^{t-1}) + (1-\beta)c^{t-1}$
Flow update	$f_r^t = f_r^{t-1} + k\sum_{(r,s)} f_r^{t-1}.[c_r^{\,t} - c_s^{\,t}]_+ \Delta_{rs}$	$f_r^t = f_r^{t-1} + \alpha[f(c^t) - f_r^{t-1}]$
Signal update	$g_I^t = g_I^{t-1} + k\sum_{(I,J)} g_I^{t-1}.[s_I.d_I^{t-1} - s_J.d_J^{t-1}]_+ \Delta_{IJ}$	$g_I^t = g_I^{t-1} + \gamma[g_{SS}(f^t) - g_I^{t-1}]$
Example network	 direct relationship between route choice and signal setting	 indirect relationship between route choice and signal setting

3 Numerical comparison

The adoption of a unifying modeling framework able to incorporate both Smith's and Cantarella's updating processes allows acquiring deeper insight into analogies and differences of the two modeling approaches, and gives opportunity to highlight strengths and limitations.

In the full paper, different numerical analyses are performed aiming to achieve the following objectives:

- To analyse the impact of flow updating rules and signal updating rules on the specification of day-to-day travelers' updating behavior (parameters), and on the equilibrium stability properties of the dynamic systems.

The route swapping rules adopted in Smith's and Cantarella's models are important determinants for the stability properties of the solutions. The rate of change of Smith's models does not essentially change with increasing t and stability is achieved by opportunely choosing the signal response function as difference in pressure between signal stages ($s \bullet d$). Under these settings, maximum throughput is shown to be achieved in the small network depicted in Table 1 (left).

In Cantarella's approach the way current information and past information are balanced is the main factor determining stability and network performance at equilibrium. Analysis of the three updating parameters α, β, and γ, shows different solution properties with different day-to-day behavioral responses when trying to minimize the total delay in the network in Table 1 (right).

Stability conditions are therefore controlled in different ways from the two approaches and question is whether a combination of the two would guarantee the same properties under weaker assumptions. The implication of stability of the dynamic signal control and route choice problem in continuous-time approach to that in discrete-time approach can be explored.

- To quantify the influence of control settings, in particular the P0 control policy, on the day-to-day dynamic processes and learning behavior.

As explained in the previous point, the choice of an opportune signal control policy may be an equally important element for guaranteeing uniqueness and stability of equilibrium. If this was demonstrated in small, design controlled toy networks, this is less straightforward in networks characterized by different topologies. In this sense the effectiveness shown for responsive policies based on the P0 policy may not be easily verified in larger networks and with different day-to-day processes and adaptation speeds.

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